Suppression of induced pseudoscalar form factor due to radiative and ordinary captures of a muon by nuclei

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The induced pseudoscalar form factor in nuclear matter is shown to be suppressed appreciably compared with the void value. This can occur in experiments on radiative μ capture by intermediate and heavy nuclei.

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Radiative capture of muons (RCM) is of particular interest to investigators because of its high sensitivity to the induced pseudoscalar form factor $g_p(q^2)$, which is one of the basic characteristics of a weak semileptonic nucleon interaction (see, for example, Refs. 1-4). A number of experiments, in which the photon spectra and energy dependences of the asymmetry coefficients of photon emission due to the capture of polarized muons by ^{40}Ca were studied, have been performed.[5-7] The results of the experiments show that the observed g_p is suppressed significantly compared with the values predicted by the hypothesis of partial conservation of axial-vector current (PCAC). The experimental data, however, are insufficiently accurate and insufficiently reliable, which is attributable primarily to the large, fast-neutron background. In addition, the analysis of experimental data did not take into account the renormalization of the form factors of weak nucleon interaction in nuclear matter, which may be rather large.[8-10] In this paper we examine the renormalization of the form factor g_p in a nucleon medium in the case of an ordinary μ capture and the radiative capture of muons. The development of the data base and the improvement of the experimental techniques will make it possible to obtain very accurate information in the near future on the characteristics of RCM by nuclei and to verify the theoretical predictions of the value of the form factor $g_p(q^2)$.

According to the PCAC hypothesis,[11] the dominant effect in the form factor $g_p(q^2)$ is that of the pion pole. The renormalization of this form factor in the nuclear matter is therefore determined by three factors: 1) the renormalization of the pion-nucleon vertex, 2) the renormalization of the pion propagator, and 3) the renormalization of the amplitude of the decay $\pi \to \mu\nu$. All three factors have been carefully analyzed in Ref. 10. If we exclude from the analysis the nonuniversal part of the renormalization, which is associated with the nucleon-nucleon hole pair production near the Fermi surface and which depends strongly on the characteristics of the initial and final states, then these factors can be taken into account in the following way. The renormalization of the πNN vertex is given by the effective nucleon charge with respect to the symmetry pole $\sigma\tau_{\pm}$, which can be represented in the form[8,9,12]

$$e_q(0) = \left[1 + \frac{1}{3}\xi\, 0{,}90\, \frac{\omega_\Delta^2}{\omega_\Delta^2 - \omega^2}\, \frac{\rho}{\rho_0}\right]^{-1}, \qquad (1)$$

where $\omega_\Delta \approx 2.4\, m_\pi$ is the energy of the $\Delta(1232)$ resonance, ρ/ρ_0 is the ratio of the density of nuclear matter to the normal nuclear density, $\rho_0 = 0.17\ \Phi^{-3}$, and the parameter ξ characterizes the degree of suppression of the Lorentz-Lorentz effect (Ref. 13): $\xi = 1$ corresponds to the unsuppressed effect and $\xi = 0$ corresponds to the absence of the effect. The experimental data are in agreement with the expression (1) at $\xi \approx 0.3$.[14,15]

The renormalization of the pion propagator in the medium is determined by the factor $D_B(\omega, \mathbf{q})\, D_0^{-1}(\omega, \mathbf{q})$, where

$$D_0 = [\omega^2 - (\mathbf{q}^2 + m_\pi^2)]^{-1}, \quad D_B = [D_0^{-1} - \Pi(\omega, \mathbf{q})]^{-1}. \qquad (2)$$

The polarization pion operator in nuclear matter $\Pi(\omega, \mathbf{q})$ at $\omega, |\mathbf{q}| \leqslant m_\pi$ can be determined in the form[8,9,12,14]

$$\Pi(\omega, \mathbf{q}) = \Pi^{(S)}(\omega) + \Pi^{(P)}(\omega, \mathbf{q}); \quad \Pi^{(S)}(\omega) = 0{,}35\, m_\pi^2 \frac{\rho}{\rho_0} - 0{,}2\, \omega^2 \frac{\rho}{\rho_0},$$

$$\Pi^{(P)}(\omega, \mathbf{q}) = \left(-0.90\, \frac{\omega_\Delta^2}{\omega_\Delta^2 - \omega^2}\, \frac{\rho}{\rho_0}\, \mathbf{q}^2\right) \Big/ \left(1 + \frac{1}{3}\xi \cdot 0{,}90\, \frac{\omega_\Delta^2}{\omega_\Delta^2 - \omega^2}\, \frac{\rho}{\rho_0}\right). \qquad (3)$$

The spatial and temporal form factors of the pion decay in nuclear matter $\tilde{f}_\pi(\omega, \mathbf{q})$ and $\tilde{f}_\pi^0$ can be expressed in terms of $\Pi^{(P)}(\omega, \mathbf{q})$ and $\Pi^{(S)}(\omega)$ in the following way[10]:

$$\tilde{f}_\pi(\omega, q) = f_\pi[1 + \Pi^{(P)}(\omega, \mathbf{q})/\mathbf{q}^2]$$
$$\tilde{f}_\pi^0(\omega, q) = f_\pi[1 - (\Pi^{(S)}(\omega) - \Pi^{(S)}(0))/\omega^2]. \qquad (4)$$

Here $f_\pi \cong 0.95\, m_\pi$ is the pion decay constant.

The spatial and time components of the induced pseudoscalar form factor in nuclear matter $\tilde{g}_P^s$ and $\tilde{g}_P^t$ are

$$\tilde{g}_P^s(q) = g_P(q^2)\, e_q(0)\, D_B\, D_0^{-1}(\omega, \mathbf{q})(\omega, \mathbf{q})\, \tilde{f}_\pi(\omega, \mathbf{q})/f_\pi,$$
$$\tilde{g}_P^t(q) = g_P(q^2)\, e_q(0)\, D_B(\omega, \mathbf{q})\, D_0^{-1}(\omega, \mathbf{q})\, \tilde{f}_\pi^0(\omega, \mathbf{q})/f_\pi. \qquad (5)$$

Since there is no Lorentz invariance in the medium, $\tilde{g}_P^t \neq \tilde{g}_P^s$, the contribution of the induced pseudoscalar to the μ-capture amplitude cannot be reduced, with the help of a Dirac equation, to an expression of the form

$$g_P[\bar{u}(n)\gamma_5 u(p)]\,\bar{u}(\nu)(1 - \gamma_5)\, u(\mu).$$

Therefore, instead of one form factor g_P in the medium, the amplitude will have two form factors $\tilde{g}_P^s$ and $\tilde{g}_P^t$, which must be multiplied by different lepton matrix elements. It is not clear in this case which of these form factors or which combination of them must be compared with the void value of g_p, and the renormalization of the induced pseudoscalar form factor in nuclear matter, strictly speaking, becomes meaningless. The problem is simplified, however, when $\alpha Z_{\text{eff}} \ll 1$, because the nonrelativistic approximation for the muon on the mesoatom orbit can be used. The lepton matrix element $\bar{u}(\nu)(1-\gamma_5)u(\mu)$ coincides with $\bar{u}(\nu)\gamma^0(1+\gamma_5)u(\mu)$ in this limit, and we can now introduce the effective value of the induced pseudoscalar form factor in nuclear matter, which is determined by the relation

$$\tilde{g}_P^{eff}(q) = \left[\frac{|\omega|}{m_\mu} \tilde{g}_P^t(q) + \frac{m_\mu - |\omega|}{m_\mu} \tilde{g}_P^s(q) \right]. \tag{6}$$

Thus the renormalization of the induced pseudoscalar in nuclear matter reduces in the case of ordinary μ capture to a replacement of g_p by $\tilde{g}_p^{\text{eff}}(q)$, where $q \equiv p - n = \nu - \mu$. It is easy to see that the expression (6) can be used in the case of RCM; the difference is in the fact that $q = q_L = p - n$ when a γ-ray quantum is emitted from the lepton part of the amplitude and $q = q_N = p - n - k$ when a γ-ray quantum is emitted from the nucleon part (k-4 photon momentum). In addition, an additional suppression due to renormalization of the electromagnetic pion vertex in the nuclear matter occurs in that part of the amplitude which corresponds to the emission of a γ-ray quantum by a virtual pion, which determines the form factor $g_p(q^2)$ (see, for example, Fig. 1 in Ref. 4). This renormalization, which can easily be determined by using the Ward identity, is

$$[\partial D_B^{-1}/\partial \mathbf{q}][\partial D_\circ^{-1}/\partial \mathbf{q}]^{-1} = 1 + \frac{\Pi^{(P)}(\omega, \mathbf{q})}{\mathbf{q}^2} = \frac{\tilde{f}_\pi(\omega, \mathbf{q})}{f_\pi}.$$

Using Eqs. (1)-(6) we can easily calculate the quantity $\tilde{g}_p^{\text{eff}}$. When $\xi = 0.3$ and the density $\rho = 0.8\,\rho_0$, which can be used for estimating the renormalization of the form factors in ^{40}Ca (Ref. 12), we obtain $\tilde{g}_p^{\text{eff}} \approx 0.45\, g_p$ for $|\omega| = 0.2\, m_\mu$ and $|\mathbf{q}| = 0.8\, m_\mu$. The spatial form factor $\tilde{g}_p^s$, which is suppressed appreciably compared with the void value, is weighted heavily and the temporal form factor $\tilde{g}_p^t$, which is suppressed to a lesser extent, is not weighted as heavily. As a result, $\tilde{g}_p^s$ and $\tilde{g}_p^t$ contribute to $\tilde{g}_p^{\text{eff}}$ approximately equally. The kinematics $|\omega| \approx 0.2\, m_\mu$, $|\mathbf{q}| \approx 0.8\, m_\mu$ correspond to the ordinary μ capture, since the average excitation energies of the nucleus $E^* = |\omega|$ lie in the region 15-25 MeV. The large values $|\omega| = E^* + E_\gamma$ are of particular interest in the case of RCM, since the induced pseudoscalar has a stronger effect in this region because of the kinematics.[1-4] At $|\omega| = 0.8\, m_\mu$ and $|q| = 0.2\, m_\mu$ we obtain $\tilde{g}_p^{\text{eff}} \approx 0.68\, g_p$; here the main contribution to $\tilde{g}_p^{\text{eff}}$ comes from $\tilde{g}_p^t$, whereas the $\tilde{g}_p^s$ contribution is very small ($\sim 4\%$). Note that the form factor $\tilde{g}_p$ was calculated in the static approximation $\omega = 0$ in Ref. 15, i.e., only the $\tilde{g}_p^s$ contribution, to which a unit weight has been assigned, was taken into account. As a result, the obtained expression for $\tilde{g}_p$ was fourfold to fivefold lower compared with our estimate of $\tilde{g}_p^{\text{eff}}$.

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