Phonon-deficit effect in superconductors in a strong microwave field

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The phonon flux from a thin superconducting film irradiated by a microwave field is derived. It is shown that in intense microwave fields, as in the case of weak fields, studied previously {A. M. Gulian (Gulyan) and G. F. Zharkov, Phys. Lett. **80A**, 79 (1980); Zh. Eksp. Teor. Fiz. **80**, 303 (1981) [Sov. Phys. JETP **53**, 154 (1981)]}, phonons are not emitted in a narrow spectral interval of phonon frequencies and are instead absorbed from the heat reservoir by the film.

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1. We have previously shown[1,2] that a thin superconducting film irradiated by a microwave field absorbs phonons from an external heat reservoir (the substrate) in a narrow spectral interval of the phonon frequencies ω_q lying near the gap, $\omega_q \gtrsim 2\Delta$. This effect which is closely related to the stimulation of superconductivity by a microwave field,[3] results from a "supercooling" of the electron subsystem. The study in Refs. 1 and 2 was applicable at low intensities of the microwave field. In the present letter we show that this phonon-deficit effect also occurs in strong fields.

Let us examine the effect on a thin superconducting film of an electromagnetic field at a frequency $\gamma \ll \omega_0 < 2\Delta$, where $\gamma = 2\tau_\epsilon^{-1}$ is the characteristic energy decay rate of the quasiparticles, and Δ is the nonequilibrium gap. We assume that the "binding energy" of the field with the electrons, α, lies in the interval ($\hbar = 1$)

$$\gamma \lesssim \alpha \ll \Delta, \tag{1}$$

where $\alpha = (e/c)^2 DA_{\omega_0} A_{-\omega_0}$, $D = 1/3\, v_F \tau_{imp}$, and A_{ω_0} is the vector potential of the field. We also assume that the film, which is in contact with a heat reservoir at the temperature T, contains a substantial number of nonmagnetic impurities, so that $\Delta\tau_{imp} \ll 1$ and $\omega_0 \tau_{imp} \ll 1$. The film is assumed to be thin enough that we can use the phonon-reservoir model[3] and thin enough for the applicability[1)] of the method of Ref. 2 for calculating the phonon emission from the film. The right-hand inequality in (1) means that the effect of the field on the density of electron levels can be ignored.[3] The left-hand inequality means two things: First, in analyzing the electron kinetics we cannot use the linearized solution of the Éliashberg kinetic equations which we used in Refs. 1 and 2 in the case $\alpha \ll \gamma$. Second, the nonequilibrium value of Δ, which is extremely sensitive to the quasiparticle distribution, differs from Δ_{BCS} and must be determined from the exact self-consistent equation. The methods for calculating and analyzing the solutions of the self-consistent equation for fields in the intensity range in (1) are described in Ref. 4. We shall use the results of that paper in the calculations of the phonon fluxes below.

2. Using the expressions from Ref. 2 for the phonon sources, and working in the linear approximation in the nonequilibrium increment $n_\epsilon^{(1)}$ ($n^{(1)} = n - n^{(0)}$), we can derive the following expression, which determines the spectral dependence of the phonon flux from the film[2)]:

$$I(\omega_q) = I_{sp}^{rel} + I_{ind}^{rel} + I_{sp}^{rec} + I_{ind}^{rec},$$

$$I_{sp}^{rel} = \int_\Delta^\infty L(\epsilon_2+\omega_q, \epsilon_2)\{-n^{(o)}_{\epsilon_2+\omega_q} n^{(1)}_{\epsilon_2} - n^{(o)}_{\epsilon_2} n^{(1)}_{\epsilon_2+\omega_q} + n^{(1)}_{\epsilon_2+\omega_q}\}\left(1 - \frac{\Delta^2}{(\epsilon_2+\omega_q)\epsilon_2}\right)d\epsilon_2,$$

$$I_{ind}^{rel} = \int_\Delta^\infty L(\epsilon_2+\omega_q, \epsilon_2)\{(n^{(1)}_{\epsilon_2+\omega_q} - n^{(1)}_{\epsilon_2}) N^{(o)}_{\omega_q}\}\left(1 - \frac{\Delta^2}{(\epsilon_2+\omega_q)\epsilon_2}\right)d\epsilon_2,$$

$$I_{sp}^{rec} = \int_\Delta^{\omega_q-\Delta} L(\epsilon_1, \omega_q-\epsilon_1)\{n^{(1)}_{\epsilon_1} n^{(o)}_{\omega_q-\epsilon_1}\}\left(1 + \frac{\Delta^2}{\epsilon_1(\omega_q-\epsilon_1)}\right)d\epsilon_1,$$

$$I_{ind}^{rec} = \int_\Delta^{\omega_q-\Delta} L(\epsilon_1, \omega_q-\epsilon_1)\{N^{(o)}_{\omega_q} n^{(1)}_{\epsilon_1}\}\left(1 + \frac{\Delta^2}{\epsilon_1(\omega_q-\epsilon_1)}\right)d\epsilon_1,$$

$$L(\epsilon_1, \epsilon_2) = \pi\lambda\frac{\omega_D}{\epsilon_F}\frac{\epsilon_1\theta(\epsilon_1^2-\Delta^2)}{(\epsilon_1^2-\Delta^2)^{1/2}}\frac{\epsilon_2\theta(\epsilon_2^2-\Delta^2)}{(\epsilon_2^2-\Delta^2)^{1/2}}, \tag{2}$$

where $n^{(0)}$ and $N^{(0)}_{\omega_q}$ are the Fermi and Bose distribution functions of the electrons and phonons, respectively, at the temperature of the external reservoir, T.

The nonequilibrium value $n^{(1)}$ in (2) must be determined from the Eliashberg kinetic equation. In those cases in which the nonequilibrium electron distribution is "smeared" over the energies in an interval significantly larger than ω_0, this equation reduces to the differential equation of Refs. 5 and 6, whose solution may be written as follows (see Ref. 4):

$$n_1^{(1)}(y) = -\frac{\Delta}{4T}\{e^{By}\int_y^\infty dx\, e^{-Bx}\phi(x) + e^{-By}[\int_0^\infty dx\, e^{-Bx}\phi(x) - \int_0^y dx\, e^{Bx}\phi(x)]\},$$

$$B = \sqrt{2\beta},\quad \beta = \frac{\gamma\Delta^2}{4a\omega_o^2} \ll 1,\quad y = \frac{(\epsilon^2-\Delta^2)^{1/2}}{\Delta}, \tag{3}$$

$$\phi(y) = \frac{y}{(2+y^2)(y^2+1)^{1/2}}\,\mathrm{ch}^{-2}\frac{\Delta(y^2+1)^{1/2}}{2T}.$$

For this solution, the total number of excitations remains constant; i.e., $\int_0^\infty n_1^{(1)}(y)\,dy=0$.

In writing (3) we have assumed that the exact function $n^{(1)}$ may be written in the form[5,6] $n^{(1)}=n_1^{(1)}+n_2^{(1)}$, where the quantity $n_2^{(1)}$ is responsible for the "warming" processes that occur in a nonequilibrium superconductor. The quantity $n_2^{(1)}$ determines the remote tail of the distribution function of the electron excitations; when it is taken into account, the total number of excitations is not conserved. In the present letter, as in Refs. 5 and 6, we shall not explicitly take $n_2^{(1)}$ into account. As Schmid *et al.*[7,8] have shown, the warming processes can be taken into account indirectly by introducing a correction term in the self-consistent equation. We write this equation as follows:

$$\int_0^{\omega_D} \mathrm{th}\,\frac{(\xi^2+\Delta^2)}{2T}\,\frac{d\xi}{(\xi^2+\Delta^2)^{1/2}} - \int_0^{\omega_D} \mathrm{th}\,\frac{\xi}{2T_{co}}\,\frac{d\xi}{\xi} - \int_\Delta^{\omega_D} \frac{n_1^{(1)}(\epsilon)\,d\epsilon}{(\epsilon^2-\Delta^2)^{1/2}}$$

$$-\frac{\pi a}{4T}\,f_1(T) - 0{,}11\,\frac{\pi}{2}\,\frac{a\omega_0^2}{\gamma_0 T_{co}}\,f_2(T) = 0. \qquad (4)$$

Here the term $-(\pi\alpha/4T)f_1(T)$ describes the dynamic suppression of the gap by the time averaged square of the field, while the last term is Schmidt's correction term.[4]

3. We have calculated the phonon fluxes from the film from Eq. (2), using $n^{(1)}$ from (3). The value of $\Delta(T)$ was found from the self-consistent equation in (4) (see Ref. 4). In general, this problem requires numerical calculations. The results of a corresponding study are shown in Figs. 1 and 2 as the spectral dependence of the phonon flux $I(\omega_q)$ (in units of $\pi\lambda\omega_D/\epsilon_F$) at $\omega_0=0.5$ and at various temperatures. (All of the quantities $\alpha, \gamma, \omega_0, T, \Delta$, and ω_q are expressed in units of T_{c0}.) These curves show that the phonon-deficit effect, which we found previously[1,2] for the case of low-intensity electromagnetic radiation ($\alpha \ll \gamma$), also occurs in the case of intense fields. The size of the effect increases with increasing frequency and amplitude of the microwave field and with increasing reservoir temperature.

In contrast with the weak-field case, which we studied analytically in Refs. 1 and 2 in the linear approximation in the field intensity, the curves obtained in the present study do not have logarithmic (fictitious) divergences at the frequency $\omega_q=\omega_0$. The

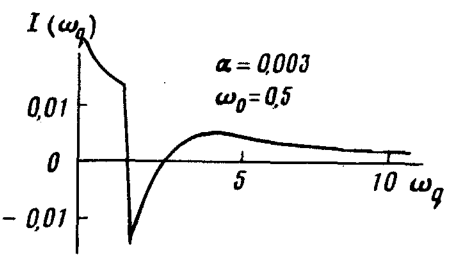

FIG. 1. Spectrum of the phonon flux from a film irradiated by a microwave field with a frequency $\omega_0=0.5$ and an intensity $\alpha=0.003$ for various temperatures: $1-T=1.02$ ($\Delta=0.25$); $2-T=0.89$ ($\Delta=1.0$); $3-T=0.5$ ($\Delta=1.7$). $\gamma=0.01$.

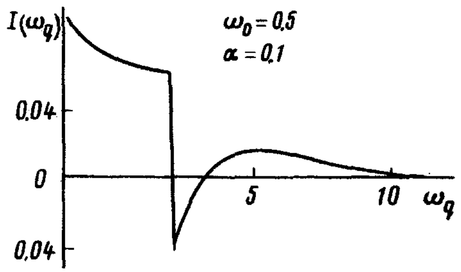

FIG. 2. The same as in Fig. 1, but for $\omega_0 = 0.5$ and $\alpha = 0.1$. $1-T = 0.79$ ($\Delta = 1.0$); $2-T = 0.7$ ($\Delta = 1.35$); $3-T = 0.5$ ($\Delta = 1.635$).

reason is that the intense field "smears out"[5,6] the nonequilibrium increment in the distribution function of electronic excitations over a broad energy interval.

As we mentioned earlier, these curves are based on solution (3), according to which the total number of quasiparticles is conserved. Since the phonon kinetics is extremely sensitive to the electron kinetics, it frequently may not be legitimate to ignore terms of the type $n_2^{(1)}$ in calculations of the phonon fluxes. This conclusion is implied, in particular, by curve 3 in Fig. 2, which lies entirely in the negative region—a result unacceptable on physical grounds. A systematic account of the "warming" processes, however, will require a special study.

Finally, we note that the phonon fluxes are very sensitive to the detailed behavior of the nonequilibrium electron distribution function because the electronic-level densities appear twice in the expressions in (2), which determine the phonon kinetics [see $L(\epsilon_1, \epsilon_2)$], while the superconducting feature appears only once in the equation for the order parameter, (4). It is for this reason that an experimental study of the nonequilibrium phonon fluxes from the superconducting film (which is interesting in its own right) might be useful for studying the nonequilibrium electronic subsystem.

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[1)]For the applicability of the theory it is sufficient to assume that d, the film thickness, is of the order of ξ_0, the correlation radius in the superconductor.

[2)]The number of phonons emitted from the film in the frequency interval d_{ω_q} is given by $dN_{\omega_q} = I(\omega_q)\,\rho(\omega_q)\,d\omega_q$, where $\rho(\omega_q) = V\omega_q^2/2\pi^2 u^3$, V is the volume of the emitting region, and u is the sound velocity.

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