Nonlinear dispersion and compression of pulses due to parametric interaction

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A parametric mechanism for simultaneous production of nonlinear dispersion of two waves that interact synchronously with the strong field of a low-frequency pump wave is discussed. The compression of pulses at the signal and idle frequencies for nonlinear dispersion is examined.

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1. In this paper we report a new parametric mechanism of pulse compression, which appears in a three-frequency synchronous interaction ($\omega_1 + \omega_2 = \omega_3$, $k_1 + k_2 \approx k_3$). Using a low-frequency monochromatic pump wave, $\omega_p = \omega_1$, we can produce in a nonlinear crystal two nonlinear-dispersion branches at the signal and idle frequencies. Using a controlled nonlinear dispersion, we can compress two pulses at

different frequencies ω_2 and ω_3 simultaneously. A number of possible nonlinear mechanisms have been discussed previously, in particular, the self-compression of wave packets during self-excitation[1-3] in a self-induced transmittance process,[4] and also by means of parametric amplification.[5] The new mechanism makes it possible to compress two PM pulses, one of which can be excited in a nonlinear crystal because of a parametric interaction; the low-frequency pump wave, which modulates the dielectric constant, propagates without distortions.

The parametric compression method can be described as follows. The interaction of signal and idle waves in the strong field of the low-frequency pumping always has the form of spatial beats even for phase synchronism, and the beat period is determined by the pumping intensity.[6] Such beats along the propagation direction can be assumed to be the result of the interference of two waves that propagate with phase velocities slightly different in magnitude. This also means that nonlinear dispersion exists in the crystal. When there is only a first-order relative linear dispersion present ($\partial k_2/\partial\omega_2 \neq \partial k_3/\partial\omega_3$), a nonlinear second-order dispersion appears in the pumping field, which can be used to compress PM pulses.

2. Let us assume that the pump wave $\mathscr{E}_1 = \frac{1}{2}(E_1 e^{i(\omega_1 t - k_1 z)} + \text{c.c.})$ modulates the dielectric constant of a crystal with a quadratic nonlinearity in accordance with the traveling-wave law. The parametric interaction of the frequency components of the signal and idle waves

$$S_j = B_j \exp\{i(\omega_j t - k_j z)\},\ j = 2, 3$$

can therefore be described by the equations

$$\frac{\partial B_2(\omega)}{\partial z} = -i\gamma_2 E_1^* B_3 e^{-i\Delta k z}, \tag{1}$$

$$\frac{\partial B_3(\omega)}{\partial z} = -i\gamma_3 E_1 B_2 e^{i\Delta k z}, \tag{2}$$

where γ_j are the nonlinearity coefficients and $\Delta k = k_1 + k_2 - k_3$. If $B_j \sim e^{-i\Gamma z}$, we can find the wave numbers $k_{\mathrm{nl},j} = k_j + \Gamma$, which have waves at the idle and signal frequencies in the nonlinear crystal,

$$k_{\mathrm{nl},2}(\omega_2) = k_2 + \frac{\Delta k}{2} \pm \sqrt{\gamma_2\gamma_3 |E_1|^2 + \frac{(\Delta k)^2}{4}}, \tag{3}$$

$$k_{\mathrm{nl},3}(\omega_3) = k_3 - \frac{\Delta k}{2} \pm \sqrt{\gamma_2\gamma_3 |E_1|^2 + \frac{(\Delta k)^2}{4}} \tag{4}$$

The dispersion of a nonlinear crystal is altered significantly because of the parametric interaction. In the presence of a relative linear dispersion two nonlinear-dispersion branches appear: One branch belongs to the in-phase components, while the other belongs to the fast counterphase components.

Following the standard dispersion theory,[7] we can determine the propagation characteristics of the wave packets by differentiating Eqs. (2) and (3) with respect to the frequencies ω_2 and ω_3 with allowance for their mutual coupling $\omega_1 + \omega_2 = \omega_3$. Calculating the first derivative, we determine the group velocity for parametric interaction

$$u_{\text{nl}}^{-1} = (\partial k_{\text{nl}j} / \partial \omega_j) = u_2^{-1} + u_3^{-1} . \tag{5}$$

It can be seen that the pulses at the signal and idle frequencies propagate with the same group velocity, slower than in a linear medium $u_{\text{nl}} < u_2, u_3$.

Furthermore, using the second derivative, we determine the nonlinear-dispersion spreading coefficient for both pulses

$$D_{\text{nl}} = (D_2 + D_3)/2 \pm \nu^2/4\gamma\,|E_1|, \tag{6}$$

where $D_j = \partial^2 k_j/\partial\omega_j^2$, $\nu = u_2^{-1} - u_3^{-1}$, and $\gamma = \sqrt{\gamma_2\gamma_3}$. A group synchronism, $u_2 = u_3$ and $\nu = 0$, has one second-order nonlinear-dispersion branch $D_{\text{nl}} = (D_2 + D_3)/2$ at each frequency.

If there is a relative first-order linear dispersion $\nu \neq 0$, then two nonlinear-dispersion branches will appear. If the group delay length $l_{\text{gr}} \approx |\Omega\nu|^{-1}$ is less than the spreading length $l_{\text{sp}} \approx |D_{\text{nl}}\Omega^2|^{-1}$, then two branches $D_{\text{nl}} \approx \pm\nu^2/4\gamma|E_1|$ can be easily isolated (here Ω is the frequency width of the pulse). The last case is of greatest practical interest. The maximum nonlinear-dispersion spreading coefficient $|D_{\text{nl}}|_{\max} \approx |\nu/\Omega|$ can be obtained with a pumping $\gamma|E_1| \approx |\Omega\nu|$.

3. The pulses behave peculiarly because of the presence of two nonlinear-dispersion branches. During propagation in a nonlinear crystal, the wave packets are divided into two approximately equal parts, one of which consists of fast components (the positive branch $D_{\text{nl}} \approx \nu^2/4\gamma|E_1|$), and the other consists of slow components (the negative branch $D_{\text{nl}} \approx -\nu^2/4\gamma|E_1|$). If a pulse with quadratic PM is directed into a parametrically excited crystal, then it will propagate in accordance with the following law:

$$E_2 = \frac{1}{2} E_{20} \left\{ \psi_+^{-1/2} \exp\left[-i\gamma E_1 z - \frac{\eta^2(1 + i\Omega_o\tau)}{\tau^2\psi_+} \right] + \psi_-^{-1/2} \exp\left[i\gamma E_1 z - \frac{\eta^2(1 + i\Omega_o\tau)}{\tau^2\psi_-} \right] \right\}, \tag{7}$$

where $\eta = t - z/u_{\text{nl}}$, $\psi_\pm = 1 \pm \Omega_0\tau z/L_{\text{sp}} \mp iz/L_{\text{sp}}$, and $L_{\text{sp}} = \tau^2/2D_\nu$ $(D_\nu = \nu^2/4\gamma|E_1|)$ is the nonlinear-dispersion spreading length.[8] The amplitude at frequency ω_3 is described by the expression in which the difference between the two parts must be written inside the braces instead of the sum.

In the absence of PM $\Omega_0 = 0$, and both parts of the pulse spread with the same velocity, but they have different phases, so that the spatial beats of the amplitudes are quasiperiodic.

In the presence of PM, one part of the pulse is compressed while the other part is decompressed, irrespective of the sign of Ω_0 (the latter is used as a pedestal for the peaking part). We recall that compression in a linear medium with one dispersion branch occurs only when the sign of the product $D_j\Omega_0$ is negative.

At the temporal focal length,

$$z_k = \frac{4\gamma E_1 |\Omega_0| \tau^3}{\nu^2 (1 + \Omega_0^2 \tau^2)} \approx \frac{2\gamma E_1 \tau}{\nu^2 |\Omega_0|} \quad (8)$$

the pulse undergoes a maximum compression, and $\tau_k \approx \Omega_0^{-1}$. If Ω_0 is varied, then pulses with a duration $\tau_k \approx 2D_\nu L/\tau$ can be produced along the length $z = L$. The limiting pulse compression provided by the parametric mechanism is $\tau_{\text{lim}} \approx |\nu|/\gamma E_1$.

4. The use of parametric interactions for compression has several positive features. First, the nonlinear dispersion can be easily controlled by varying the pumping intensity. Second, the compression of PM pulses occurs regardless of the sign of the initial frequency chirp. Third, two short pulses are formed simultaneously at the signal and idle frequencies.

This mechanism can be realized in optics. At a pumping that provides $(\gamma|E_1|)^{-1} = 1$ mm in a crystal with $\nu = 10^{-2} c^{-1}$ (c is the velocity of light) we can obtain a large dispersion $D_\nu \approx 10^{-24}$ sec^2 $\cdot$ cm^{-1}. Under such conditions, light pulses can be formed with durations down to $\tau = 10^{-13}$ sec.

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