$P \to l^+ l^-$ decays in a nonlocal quark model

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The relative widths of pseudoscalar-meson decay into lepton pairs in a nonlocal quark model have been calculated. The obtained results are in agreement with the available experimental data.

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The decays of strongly interacting particles into leptons are a valuable source of information on the electromagnetic structure of hadrons, since the leptons do not participate in strong interactions; because of this, the contribution from an electromagnetic interaction can by identified in the processes in which they participate.

Recent experiments at CERN (Ref. 1) and Serpukhov (Ref. 2), in which the ratios $B_\gamma(\pi^0 \to e^+e^-) = \Gamma(\pi^0 \to e^+e^-)/\Gamma(\pi^0 \to \gamma\gamma)$ and $B_\gamma(\eta \to \mu^+\mu^-) = \Gamma(\eta \to \mu^+\mu^-)/\Gamma(\eta \to \gamma\gamma)$ were measured, have stimulated an attempt to describe them theoretically in various strong-interaction models.

The form factor of the decay $P \to \gamma\gamma$

$$M(P \to \gamma\beta) = e^2 g_{P\gamma\gamma} F(k_1^2, k_2^2)\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu(k_1)k_1^\nu\epsilon^\alpha(k_2)k_2^\beta,$$

$$F(0, 0) = 1.$$

plays the main role in the theoretical descriptions of the decays $P \to l^+l^-$. Note that the diagram, which describes the decay $P \to l^+l^-$, diverges logarithmically in the case $F = \text{const}$.

In the initial studies[3,4] the form factor was chosen from the start in such a way that the appropriate integral would converge. The result in this case depended on the arbitrary cutoff parameter.

In Refs. 5 and 6 the process $P \to l^+l^-$ was analyzed in terms of the vector-dominance model (VDM). The cutoff parameter, however, had to be used even in this case.

It was assumed in Ref. 7 that the $P \to \gamma\gamma$ decay proceeds via the baryon loop. The final result depended on the mass of the intermediate baryon.

In these studies it was possible to establish the lower limit of the ratio B_γ, which is determined by the model-independent, absorptive part of the amplitude (the so-called unitary limit), and also to investigate the influence of the real part, which depends substantially on the choice of the model.

In Ref. 8 the $P \to l^+l^-$ decay was calculated in the model of the triangular quark anomalies dominance (TQAD). The calculation was performed with the form factor

$$F(k_1^2, k_2^2) = \frac{m_V^2}{m_V^2 - k_1^2} \frac{m_V^2}{m_V^2 - k_2^2},$$

where V are the vector mesons.

The hypotheses and assumptions, which were used specifically to describe the $P \to l^+ l^-$ decay, are the common feature of these procedures.

In this letter we analyze the $P \to l^+ l^-$ decays in the nonlocal quark model (NQM), which is a self-consistent relativistic scheme of the quantum field bag.[9] Using this model with only two parameters that characterize the quark field, we were able to describe a broad range of hadronic decays and, specifically, to determine subtle characteristics such as the form factors of the decays $P \to \gamma l^+ l^-$ (Ref. 10) and $\omega \to \pi^0 \mu^+ \mu^-$ (Ref. 11). Since all the Feynman diagrams in the nonlocal quark model are similar, we found it worthwhile to calculate the $P \to l^+ l^-$ decay.

The $P \to l^+ l^-$ decay is illustrated diagrammatically in Fig. 1. Accordingly, the invariant amplitude can be written in the form

$$M(P \to l^+ l^-) = \lim_{\delta \to 0} 4\pi\alpha \int \frac{d^4 k}{(2\pi)^4 i} \epsilon_{\mu\rho\sigma\nu} p^\rho k^\sigma e^2 g_{P\gamma\gamma}$$

$$\times F^\delta\left(-(p - k)^2, -k^2\right) \frac{\bar{v}(p_-)\gamma^\mu (m_l + \hat{k} - \hat{p}_+)\gamma^\nu v(p_+)}{k^2 (k-p)^2 [m_l^2 - (k - p_+)^2]}, \quad (1)$$

where δ is the regularization parameter.[9] The quantity $g_{P\gamma\gamma}$ has been determined explicitly in Ref. 11.

The form factor F, which determines the amplitude of the decay $P \to \gamma\gamma$, has the form

$$F(k_1^2, k_2^2) = \lim_{\delta \to 0} F^\delta(k_1^2, k_2^2) = 2 \iiint_0^1 d^3\alpha\, \delta\left(1 - \sum_{i=1}^{3} \alpha_i\right) A[\alpha_1\alpha_2 k_1^2$$

$$+ \alpha_2\alpha_3 k_2^2 + \alpha_1\alpha_3 p^2], \quad (2)$$

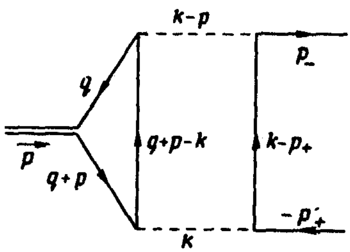

FIG. 1.

where $A(t) = \exp(-t) \cos \xi \sqrt{t}$.

The integral (1) is calculated in the following way.

1. We shall switch to the Euclidean metrics $k_0 \to ik_4$ and remove the regularization $\delta \to 0$. The obtained integral converges because the function $F(k_1{}^2, k_2{}^2)$ decreases in the Euclidean region.

2. We shall use the inverse Mellin transformation for the $A(t)$ function[12]

$$A(t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} ds\, t^{-s} \widetilde{A}(s), \quad \sigma > 0$$

$$\widetilde{A}(s) = \int_0^\infty dt\, t^{s-1} A(t).$$

3. Using the double Mellin representation

$$\frac{1}{(a + x + y)^s} = \frac{1}{2i} \int_{-\gamma_1 + i\infty}^{-\gamma_1 - i\infty} \frac{dz_1}{\sin \pi z_1} \frac{1}{2i} \int_{-\gamma_2 + i\infty}^{-\gamma_2 - i\infty} \frac{dz_2}{\sin \pi z_2}$$

$$\times \frac{\Gamma(s + z_1 + z_2)}{\Gamma(1 + z_1)\,\Gamma(1 + z_2)\,\Gamma(s)} \frac{x^{z_1} y^{z_2}}{a^{s + z_1 + z_2}},$$

$$\gamma_i > 0, \quad \mathrm{Re}\, s = \sigma > 0, \quad \mathrm{Re}\,(s + z_1 + z_2) = \sigma - \gamma_1 - \gamma_2 > 0$$

for the expression $[\alpha_1 \alpha_2 (p-k)^2 + \alpha_2 \alpha_3 k^2 + \alpha_1 \alpha_3 p^2]^{-s}$, we shall integrate over the parameters α.

4. Integration over k is performed with the help of Feynman parametrization. Integration over the Feynman parameters is carried out by using the Mellin representation

$$\frac{1}{[p^2 u_1 u_2 + m_l^2 u_3^2]^{-z_1 - z_2}} = \frac{1}{(p^2)^{-z_1 - z_2}} \frac{1}{\Gamma(-z_1 - z_2)} \frac{1}{2i} \int_{-\gamma_3 + i\infty}^{-\gamma_3 - i\infty}$$

$$\times \frac{dz_3}{\sin \pi z_3} \frac{1\,(z_3 - z_1 - z_2)}{\Gamma(1 + z_3)} \left(\frac{m_l^2 u_3^2}{p^2 u_1 u_2} \right)^{z_3} (u_1 u_2)^{z_1 + z_2}$$

$$0 < \gamma_3 < -\mathrm{Re}\,(z_1 + z_2).$$

5. We shift the paths in the fourfold contour integral in such a way that we obtain terms in the parameters $\mu_p{}^2 = m_p{}^2 L^2/4$ and $m_e{}^2/m_p{}^2$.

Taking into account only the main terms in these parameters, we find

$$M(P\to l^+l^-) = a^2 g_{P\gamma\gamma}\epsilon_{\mu\rho\nu\sigma}p^{\rho}\bar{v}(p_-)\gamma^{\mu}\gamma_a\gamma^{\nu}v(p_+)$$

$$\times\left[\frac{1}{4}g_{\sigma a}I - \frac{p_+^{\sigma}p_+^{a}}{p^2}N\right],$$

where

$$I = (\ln\mu_P^2 - \frac{7}{2} - \widetilde{B}'(0)) - i\pi,$$

$$N = (2\ln^2\frac{m_l}{m_P} + 6\ln\frac{m_l}{m_P} + 3) - i\pi(2\ln\frac{m_l}{m_P} + 3);$$

$$\widetilde{B}(s) = s(s+1)\widetilde{A}(s),$$

$$\widetilde{B}'(0) = 1 + 4\int_0^{\infty} du\, \exp(-u^2)[u\cos\xi u + \frac{1}{2}\xi\sin\xi u]\ln u \approx -0.416.$$

Calculating the decay width in a standard way, we finally obtain

$$B_{\gamma}(P\to l^+l^-) = \Gamma(P\to l^+l^-)/\Gamma(P\to\gamma\gamma)$$

$$= \frac{1}{2}\left(\frac{a\, m_l}{\pi\, m_P}\right)^2 \{9|I|^2 + \beta^4|N|^2$$

$$+ 6\beta^2(\mathrm{Re}\, I\ \mathrm{Re}\, N - \mathrm{Im}\, I\ \mathrm{Im}\, N\} \times \beta,$$

$$\beta = \sqrt{1 - 4m_l^2/m_P^2}.$$

TABLE I.

$\Gamma(P\to l^+l^-)/\Gamma(P\to\gamma\gamma)$	Experiment	Unitary limit [3 –6]	TQAD [8]	NQM
$\pi^0\to e^+e^-$	$22^{+27}_{-11}\times10^{-8}$ (Ref. 1)	$4,58\times10^{-8}$	5.9×10^{-8}	$5,38\times10^{-8}$
$\eta\to\mu^+\mu^-$	$(1,7\pm0,5)\times10^{-5}$ (Ref. 2)	$1,20\times10^{-5}$	$1,24\times10^{-5}$	$2,38\times10^{-5}$
$\eta\to e^+e^-$	—	$4,32\times10^{-9}$	$13,2\times10^{-9}$	$9,97\times10^{-9}$
$\eta'\to\mu^+\mu^-$	—	$4,06\times10^{-6}$	$6,1\times10^{-6}$	$8,1\times10^{-6}$
$\eta'\to e^+e^-$	—	$1,06\times10^{-9}$	8.1×10^{-9}	$4,9\times10^{-9}$

The numerical values of $B_\gamma(P \to l^+ l^-)$ are given in Table I. We can see that these results are in a reasonable agreement with the experiment. For comparison, we give the results of Refs. 3–6 and 8.

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