# Contour equation for supersymmetric gauge theory

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A contour equation for generalization of the Wilson Loop to the supersymmetric gauge theory is derived.

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A contour treatment[1-3] of the non-Abelian gauge field theories involves, in particular, a reformulation of the classical and quantum equations of motion in terms of the nonlocal value

$$u(C_{xy}) = P\exp\Big(\int_{C_{xy}} A_m dx^m\Big) \; , \qquad (1)$$

  

where $C_{xy}$ is a curve with the origin at the point $x$ and the end at the point $y$. In this letter we shall derive a contour equation, which is analogous to the Makenko-Migdal equation, for the supersymmetric gauge theory, i.e., for supersymmetric generalization of the value $u(C)$. The supersymmetric generalization was examined in Refs. 4 and 5, and in Ref. 5 it was used for a superfield derivation of the supercurrent anomaly in the supersymmetric QED. The supersymmetric contour equation, which is different from ours, was proposed in Ref. 6 for another (Abelian) model—the Ogievetskiĭ-Sokachev model.[7]

Below we analyze the standard supersymmetric gauge theory ($N = 1$) with a non-Abelian gauge group.[8]

The contour in the superspace is the set $z^M(s) = x^m(s), \theta_\mu(s), \theta_{\dot\mu}(s)$, where $\{x^m(s)\}$ is a contour in the ordinary space, and $\theta_\mu, \overline{\theta}_{\dot\mu}$ of each $s$ are independent generatrices of the infinite-dimensional Grassman algebra with an involution (for which $\theta_\mu \leftrightarrow \overline{\theta}_{\dot\mu}$).[9] The supersymmetry-supertranslation transformations of the contour have the following form:

$$C \to C_T = \{ x^m + i\theta\sigma^m\overline{\eta} - i\eta\sigma^m\overline{\theta},\ \theta + \eta,\ \overline{\theta} + \overline{\eta} \},$$

$$(\sigma^m)_{\alpha\dot\beta} = (1, \vec{\sigma})_{\alpha\dot\beta} (\vec{\sigma}^m)^{\dot\alpha\beta} = (1, -\vec{\sigma})^{\dot\alpha\beta} \tag{2}$$

where $\vec{\sigma}$ are the Pauli matrices, and $\eta$ is a constant ($s$-independent) spinor.

The generalization of $u(C)$ for the contours in the superspace is given by the formula[5]

$$U(C_{z_1 z_2}) = P\exp\Big(\int_{C_{z_1 z_2}} ds\, \dot{z}^M A_M\Big), \quad \mathrm{Tr}\, U(C_{zz}) \equiv \psi(C), \quad \dot{z}^M = \frac{dz^M}{ds}. \tag{3}$$

Here $A_M$ are the superpotentials,[8] which are transformed as a "convecter" in the supertranslations (i.e., in a contravariant manner with respect to $\dot{z}^M$). The superpotentials $A_A$, which are superfields (i.e., their argument is replaced as a result of supertranslations), are derived from $A_M$ after multiplication by a tetrad[8]

$$A_A = e^M_A A_M, \quad e^M_A = \begin{pmatrix} \delta^m_a & 0 & 0 \\ i(\sigma^m\overline{\theta})_\alpha & \delta^\mu_\alpha & 0 \\ i(\theta\sigma^m)_{\dot\beta}\epsilon^{\dot\beta\dot\alpha} & 0 & \delta^{\dot\alpha}_{\dot\mu} \end{pmatrix}, \quad \epsilon^{\dot\alpha\dot\beta} = -\epsilon^{\dot\beta\dot\alpha} = \epsilon_{\dot\beta\dot\alpha}, \quad \epsilon^{10} = 1. \tag{4}$$

Operations on the contour functionals—the derivative with respect to the small area $\delta/\delta\sigma^{MN}$ and the covariant derivative $\partial_M$—can be easily applied to the case of contours in superspace if we use their definition[2] in terms of the variational derivatives $\delta/\delta\dot{z}^M(s)$. If $M$ is a spinor index, then this must be a backward ferionic variational derivative.[9] The standard formulas apply in this case

$$\frac{\delta\psi(C)}{\delta\sigma^{MN}(z)} = \mathrm{Tr}\, F_{MN} U(C_{zz}), \quad F_{MN} = \left(\frac{\partial}{\partial z^M} A_N + A_M A_N\right) \pm (M \leftrightarrow N), \tag{5}$$

$$\partial_K \frac{\delta}{\delta \sigma^{MN}} \psi(c) = \mathrm{Tr}\, D_K F_{MN} U(C_{zz}), \qquad D_K = \frac{\partial}{\partial z^K} + [A_K, \quad ]_\pm . \tag{6}$$

The derivatives with the indices $A, B\ldots$ are obtained, as usual, by substituting $e_A^M \delta/\delta z^M$ for $\delta/\delta z^M$ in the definitions, The supersymmetric gauge theory with $N=1$ is formulated[8] by reducing the superpotentials $A_A$ to the Hermitian matrix superfield $V(x\theta\bar{\theta})$

$$A_{\dot{\alpha}} = 0, \quad A_\alpha = e^{-V} D_\alpha e^V, \qquad A_a = \frac{i}{4} \bar{\sigma}_a^{\dot{\alpha}\beta} \bar{D}_{\dot{\alpha}} A_\beta . \tag{7}$$

The differentiation operators on the right-hand side of Eq. (7) lead to the appearance of the $\nabla_A$ operator on the right-hand side of the contour equation (9).

The action[8] gives rise to the equations of motion

$$\bar{D}_{\dot{\alpha}} (\bar{\sigma}^a)^{\dot{\alpha}\beta} F_{\beta a} = 0. \tag{8}$$

By writing the equations of motion in the geometric form (8), we can represent them as an equation for $\psi(C)$ with the help of Eq. (6); this gives rise to the operator on the left-hand side of the contour equation (9).

Proceeding in a manner analogous to the original derivation of the MM equation,[3] we obtain a set of equations for the "Green's contour functions" $\langle \psi(C_1) \ldots \psi(C_p) \rangle$. Because these equations are cumbersome, we shall omit them as well as their derivation; however, we shall give their corollary—a closed equation for $\omega(C) = \langle 1/n\, \psi(C) \rangle$ in the limit of the infinite number of colors ($n \to \infty$)

$$\bar{D}_{\dot{\alpha}} (\bar{\sigma}^a)^{\dot{\alpha}\beta} \frac{\delta \omega(C)}{\delta \sigma^{\beta a}(z)} = \lambda \int_C ds \dot{z}_1^A \nabla_A (\omega(C_{z_1 z}) \omega(C_{z z_1}) \delta(z_1 - z)),$$

$$\nabla A = \left[ \frac{i}{4} \bar{\sigma}_a^{\dot{\alpha}\beta} \left( (1-\alpha) \bar{D}_{\dot{\alpha}} D_\beta - \alpha D_\beta \bar{D}_{\dot{\alpha}} \right), \ (1-\alpha) D_\alpha, \quad -\alpha \bar{D}_{\dot{\alpha}} \right] \tag{9}$$

$\lambda$ is the coupling constant.

Because of the equality $\int_C ds \dot{z}^M \partial/\partial z^M = 0$, the right-hand side of Eq. (9) is independent of the arbitrary parameter $\alpha$. This equation must be complemented by the equations corresponding to Bianchi's identities.[8] We shall write only one equation which shows that the choice of the operator on the right-hand side of Eq. (9) is ambiguous,

$$\bar{D}_{\dot{\alpha}} (\bar{\sigma}^a)^{\dot{\alpha}\beta} \frac{\delta \omega(C)}{\delta \sigma^{\beta a}} + D_\alpha (\bar{\sigma}^a)^{\dot{\beta}\alpha} \frac{\delta \omega(C)}{\delta \sigma^{\alpha a}} = 0 . \tag{10}$$

It is easy to verify that Eq. (9) is invariant in the supertranslations: if $\omega(C)$ is a solution, then $\omega(C_T)$ must also be a solution. This property generalizes the properties of the MM equation (Ref. 3) in the standard translations.

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