Heat transfer in a turbulent laser plasma

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Gas-dynamic equations are formulated for turbulent plasma flows in which ion-acoustic turbulence is induced by an intense electron heat flux.

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Several pieces of evidence indicate that experimental results on the flow of laser plasmas contradict the theory of laminar plasma flows. Furthermore, it now seems settled that there is an anomalous suppression of heat transfer under the conditions corresponding to these contradictions.[1–6] Bickerton stated some time ago[7] (see also Refs. 8 and 9) that the suppression of heat transfer might be caused by the onset of turbulence in the plasma as a result of an ion-acoustic instability driven by an electron heat flux.[10] Research on turbulent flows of laser plasmas has been delayed by the lack of analytic equations to relate the heat flux to the temperature gradient in a turbulent laser plasma. In turn, the derivation of this theory is being held up primarily by the primitive state of the theory for the ion-acoustic turbulence of plasmas, in which the fluctuation distribution has been found only with respect to the magnitude of the wave vectors,[11] and the angular distribution remains an open question.[11,12]

In this letter we will formulate the basic equations for the turbulent flow of a nonisothermal, collisionless plasma, working from an angular distribution which we have found for the turbulent fluctuations.

The physics of a turbulent plasma can be illustrated by the kinetic equation for the number of ion-acoustic waves, $N(\mathbf{k}, \mathbf{r}, t)$:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\left(\frac{\partial \omega}{\partial \mathbf{k}} N\right) = -2\frac{\delta\epsilon_e''(\omega,\mathbf{k})}{[\partial\epsilon'/\partial\omega]} N - \frac{\kappa T_i N}{m_i n_i e_i^2} \int \frac{d\mathbf{k}}{(2\pi)^4}[\mathbf{k}\mathbf{k}']^2$$

$$\times \frac{(\mathbf{k}\mathbf{k}')^2}{(kk')^2}\delta\epsilon_i''(\omega-\omega', \mathbf{k}-\mathbf{k}')\, N(\mathbf{k}', \mathbf{r}, t) \qquad (1)$$

where

$$\delta\epsilon_{e(i)} = \frac{4\pi e_{(i)}^2}{k^2 m_{e(i)}} \int \frac{d\mathbf{v}}{\omega - \mathbf{k}\mathbf{v}}\left(\mathbf{k}\frac{\partial f_{e(i)}}{\partial \mathbf{v}}\right) = \delta\epsilon'_{e(i)} + i\delta\epsilon''_{e(i)},$$

$$\epsilon' = 1 + \delta\epsilon_e' + \delta\epsilon_i',$$
$$\omega = \omega(\mathbf{k}),\ \omega' = \omega(\mathbf{k}').$$

This equation describes the Čerenkov interaction of waves with electrons and the induced scattering of waves by ions. Correspondingly, in taking these interactions into account we should use kinetic equations for the electron and ion distribution functions.

To describe the transport in a turbulent plasma, we use the Grad moment method, writing the distribution functions in the form

$$f_{e(i)} \frac{n_{e(i)} m_{e(i)}^{3/2}}{[2\pi\kappa T_{e(i)}]^{3/2}} \exp\left[-\frac{m_{e(i)}(\mathbf{v}-\mathbf{u}_{e(i)})^2}{2\kappa T_{e(i)}}\right]\left\{1+\frac{m_{e(i)}\mathbf{q}_{e(i)}(\mathbf{v}-\mathbf{u}_{e(i)})}{n_{e(i)}\kappa^2 T_{e(i)}^2}\right.$$
$$\left.\times\left[\frac{m_{e(i)}(\mathbf{v}-\mathbf{u}_{e(i)})^2}{5\kappa T_{e(i)}}-1\right]\right\}. \qquad (2)$$

We will restrict the discussion to a quasineutral ($n_e = zn_i$) plasma without a current, with $\mathbf{u}_e = \mathbf{u}_i$, and we will assume that the plasma as a whole is flowing at the velocity $\mathbf{u} \approx \mathbf{u}_i$. The ion-acoustic frequency is then $\omega(\mathbf{k}) = kv_s + \mathbf{ku}$, where v_s is the ion-acoustic velocity.

For the hydrodynamic flows in which we are interested here, with $zT_e/T_i \ll (m_i/m_e)^{1/2}$, the left side of Eq. (1) is negligibly small. We then find

$$N(\mathbf{k}) = N(k,\theta) \approx \frac{26\, q_e}{\kappa T_i}\,\frac{(m_e m_i)^{1/2}}{k^4}\,\ln\left(\frac{1}{kr_{De}}\right)\delta(\cos\theta - \cos\theta_0), \qquad (3)$$

where θ is the angle between the wave vector $\mathbf{k}$ and the electron heat flux $\mathbf{q}_e$. The dependence of N on the wave number corresponds to the Kadomtsev-Petviashvili spectrum,[11] and the angular distribution of the ion-acoustic turbulence is anisotropic, with a "jet" at the angle $\theta_0 \approx 145°$. The steady turbulent state, with the ion-acoustic noise level in (3), sets in at $q_e \gtrsim 6.5 n_e \kappa T_e v_s$.

According to (3), the transport equations corresponding to (2) have the form

$$\partial n_e/\partial t + \mathrm{div}\, n_e\mathbf{u} = 0, \quad m_i n_i\left(\partial\mathbf{u}/\partial t + (\mathbf{u}\nabla)\mathbf{u}\right) + \nabla n_e \kappa T_e = 0, \qquad (4)$$

$$\nabla\kappa T_e = -0.3\,\frac{m_e r_{De}}{r_{Di}^2}\, q_e \mathbf{q}_e/(n_e\kappa T_e)^2, \qquad (5)$$

$$\partial T_e/\partial t + [\mathbf{u} - v_s\mathbf{q}_e/q_e]\,\nabla T_e + (2/3)T_e\,\mathrm{div}\mathbf{u} + (2/3n_e\kappa)\,\mathrm{div}\mathbf{q}_e = 0, \qquad (6)$$

$$\partial T_i/\partial t + \mathbf{u}\nabla T_i + zv_s(\mathbf{q}_e/q_e)\nabla T_e + (2/3)T_i\,\mathrm{div}\mathbf{u} = 0, \qquad (7)$$

where $r_{De(i)}$ is the electron (or ion) Debye length. In deriving the transport equations we ignored the ion heat flux $\mathbf{q}_i$, as we are justified in doing by virtue of the inequality $v_s \gg \sqrt{\kappa T_i/m_i}$. Equations (6) and (7) differ from the ordinary hydrodynamic equations in that they contain terms $v_s(\mathbf{q}_e/q_e)\nabla T_e$ and $zv_s(\mathbf{q}_e/q_e)\nabla T_e$. Equation (5) is qualitatively different from the ordinary linear relationship $\mathbf{q}_e = -\lambda \nabla T_e$ between the heat flux and the temperature gradient. On this basis we may assert, in particular, that these results establish an inverse proportionality between the thermal conductivity and the magnitude of the turbulent heat flux density, $\lambda \approx 3\kappa^3 n_i n_e T_i T_e r_{De}/m_e q_e$, for a plasma with well-developed ion-acoustic turbulence. This relationship corresponds to a decrease in the effective mean free path of the electrons with increasing intensity of the turbulence fluctuations, according to Eq. (3).

A qualitative change in the heat-conduction equation, (5), leads to a characteristic dimension for the changes in the temperature and the other hydrodynamic quantities which is much smaller than in the ordinary laminar theory. Specifically, from (5) we find this characteristic dimension to be $L \lesssim (m_i/m_e)(T_i/T_e) r_{De}$. In a laser plasma, this estimate gives, in order of magnitude, the dimension of the region in which the dense plasma is heated, from the critical-density surface (where the laser energy is evolved) to the ablation front. With $n_e \approx 10^{21}$ cm^{-3}, $T_e/T_i \sim 10$, and $T_e \sim 1$ keV, this dimension is estimated to be $L \lesssim 10^{-4}$ cm.

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