Plasmon instability in semiconductors with parametric cyclotron pumping

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A new effect is predicted: an instability of longitudinal plasma waves at a parametric resonance in semiconductors. The growth rates are derived. The conditions under which the effect might be observed experimentally are discussed.

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A parametric resonance in a semiconductor in a time-modulated homogeneous magnetic field **H**,

$$H(t) = H_0(1 + a\cos\gamma t), \qquad a \ll 1, \quad \mathbf{H}(t) \parallel 0z, \tag{1}$$

was predicted in Ref. 1. The resonance occurs in the electron system because of a modulation of the frequency of the collective cyclotron rotation in the magnetic field in (1) and in an inhomogeneous vortical electric field at the frequency γ, $(E_\sim = \dot{H}y/c,\ \mathbf{E}_\sim \parallel 0x)$. This effect is a parametric-resonance instability of the same nature as the ordinary parametric resonance in a mechanical system. The only nonlinear mechanism which operates to stop this instability is that due to the nonquadratic nature of the electron dispersion law in the semiconductor. It was found in Ref. 1 that the steady-state amplitude and phases of the waves do not depend on the initial conditions, so that the following distribution function is established in the electron system:

$$F = \frac{N}{2(2\pi mT)^{1/2}} \sum_{i=1}^{2} \delta[p_x + m\Omega_0 \xi_i(t)]\delta[p_y - m\dot{\xi}_i(t)]\exp(-p_z^2/2mT). \tag{2}$$

Here N is the density, m is the mass, $\Omega_0 = eH_0/mc$ is the cyclotron frequency, T is the temperature, **p** is the momentum, $\xi_i(t) = A\cos(\gamma t/2 + \nu_i)$ is the y coordinate of the electrons, the dot denotes the derivative with respect to t, and A and θ_i are the amplitude and phases of the steady-state waves:

$$A^2 = \eta\Theta(\eta), \qquad \eta = |\mu_3|^{-1}[(\mu_1^2 - \nu^2)^{1/2} - \mu_2 \operatorname{sgn}\mu_3],$$

$$2\theta_1 = -\arcsin(\nu/\mu_1)\operatorname{sgn}\mu_3 + \pi\Theta(\mu_3), \quad \theta_2 = \theta_1 + \pi, \tag{3}$$

$$\mu_1 = a\Omega_0^2/\gamma, \qquad \mu_2 = \Omega_0 - \gamma/2, \quad \mu_3 = 8m^3\Omega_0^4\delta/\gamma.$$

Here $\Theta(x)$ is the unit step function, sgn$x$ means the sign of x, ν is the pulsed frequency of collisions with volume scatterers, and δ is the nonquadratic parameter $[\epsilon(\mathbf{p}) = p^2/2m + \delta(p_x^2 + p_y^2)^2]$.

Distribution (2) is a nonequilibrium, anisotropic distribution, so that an instability of longitudinal (Langmuir) plasmons occurs in the semiconductor. The growth rate of the instability can be found from the dispersion relation, which is in turn found from the vanishing of the current $\epsilon E + 4\pi j$ ($\mathbf{E} \parallel \mathbf{j} \parallel \mathbf{k} \parallel 0x$, $\mathbf{k}$ is the wave vector, and ϵ is the static dielectric constant). The current density j is calculated in the customary way from a kinetic equation which is linearized with respect to the function in (2) for the plasmon field, $E(x, t) = \exp(ikx)E(t)$. As a result, we find the equation

$$\frac{\partial E}{\partial t} + 2q\nu_1\omega_p^2 \frac{\partial}{\partial \nu_1} \int_{-\infty}^{t} dt' E(t') e^{\nu_1(t'-t)} \cos\left(\frac{\gamma t}{2} + \theta_1\right)\cos\left(\frac{\gamma t'}{2} + \theta_1\right) \times \cos\left\{kA\left[\sin\left(\frac{\gamma t'}{2} + \theta_1\right) - \sin\left(\frac{\gamma t}{2} + \theta_1\right)\right]\right\}. \qquad (4)$$

Here $\omega_p = (4\pi Ne^2/m\epsilon)^{1/2}$ is the plasma frequency, $\nu_1 = \nu_0(m\Omega_0^2 A^2/2T)^q$ is the pulsed relaxation frequency of the parametrically pumped electrons with the energy $m\Omega_0^2 A^2/2$, and q is the exponent in the energy dependence $\nu(\epsilon)$. From the integro-differential equation in (4) in the low-frequency limit $[(\omega_p^2\nu_1)^{1/3} \ll \gamma/2]$ we can easily find the growth rate of the longitudinal field $E(t)$ by taking the average of the integral operator over the "fast time. For long waves ($kA \ll 1$) we find

$$\omega = -iq\nu_1\left(\frac{2\omega_p}{\gamma}\right)^2\left[1 - \frac{3}{8}(kA)^2\right]. \qquad (5)$$

In the case[1] $q < 0$, the longitudinal plasmon is absolutely unstable. For $kA \gg 1$,

$$\omega = -iq\nu_1\left(\frac{2\omega_p}{\gamma}\right)^2 \frac{2}{(kA)^2}\left[1 - \frac{1 + \sin(2kA)}{\pi kA}\right]. \qquad (6)$$

Again in this case the plasmon is unstable at $q < 0$, although the growth rate is smaller. In this case we should also observe "geometric" oscillations with a relatively small amplitude.

In the high-frequency limit $[(\omega_p^2\nu_1)^{1/3} \gg \gamma/2]$ with $kA\gamma/2 \ll |\omega|$ we find the dispersion relation

$$\omega^3 = -iq\nu_1\omega_p^2, \qquad (7)$$

from which it follows that the plasmon is unstable not only at $q < 0$ but also at[2] $q > 0$. At large values of $kA\gamma/2\omega$ the resonant frequency of the plasmon is determined by Eq. (6) without the second term in the brackets.

The steady-state oscillation of distribution function (2) over time, at a frequency $\gamma/2$, also leads to the excitation of longitudinal electric waves in the electron system with a discrete frequency spectrum $n\gamma/2$ ($n = 1, 2, 3, \ldots$). With parametric-resonance pumping, these stimulated oscillations grow at a rate $|\mathrm{Im}\,\omega| \ll \gamma/2$. To save space here, we write out the instability condition for only the first harmonic of $\gamma/2$:

$$q(2\omega_p/\gamma)^2 \psi(kA) > 1, \qquad (8)$$

where the function $\psi(z)$ has the asymptotic behavior $\psi(z) \approx 3/4$ at $z \ll 1$ and $\psi(z) \approx -2/z^2$ at large values of z. We see that for this scattering mechanism (for the given sign of q) the wave growth rate changes sign at $kA \sim 1$.

To the best of our knowledge, these effects have not been observed experimentally. It would apparently be possible to observe them in pure indium antimonide samples at low temperatures. For the typical parameter values for InSb [$N = (1\text{–}30) \times 10^{14}$ cm^{-3}, $\epsilon = 16.8$, and $m = 0.0134 m_0$; Ref. 2], at a collision frequency $\nu \sim 10^{10}$ s^{-1}, the parametric resonance and the plasmon instability should be observable at frequencies $\gamma/2 \approx \Omega_0 \approx (0.9 + 1.3) \cdot 10^{12}$ s^{-1} in fields H_0 of the order of a few kiloersteds and at modulation amplitudes $\alpha H_0 \approx 20\text{–}100$ Oe. The power level of the cyclotron parametric pumping would be between a few tenths of a kilowatt and a few kilowatts, so that pulsed operation would probably be necessary.

1) For example, in the case of Coulomb scattering by charged impurities we would have $q = -3/2$.
2) At $q > 0$ there is a certain value $k_0 \sim (\omega_p^2 q \nu_1)^{1/3}/A\gamma$ at which the growth rate vanishes.

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