Mass formula in the restoration scheme of the Wigner SU(4) symmetry in heavy nuclei

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The contribution of higher-order Casimir operators of the SU(4) group to the mass formula in the nuclei with $A \geq 216$ is investigated on the basis of the experimental data for nuclear-mass differences. It was found that the mass differences of isobaric nuclei are described in the restoration scheme of the SU(4)) symmetry with an accuracy of ~ 200 keV.

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The detection of a spin-flip resonance in charge-exchange reactions[1-3] and its similarity to an analogous resonance in heavy nuclei[4] seems to confirm the hypothesis, advanced in the study of the mass ratios of intermediate and heavy nuclei on the basis of the Wigner SU(4) symmetry,[5] that this SU(4) symmetry is restored with the increase of the N-Z neutron excess. The assumption that the SU(4) symmetry can be restored makes it possible to refine the structure of the symmetry term in the mass formula. The first mass formula in the SU(4) scheme was proposed by Wigner[6] in the form

$$M(A, T_Z) = a(A) + b(A)\frac{1}{2}(t^2 + s^2 + y^2 - 5) + V_C(A, Z) \tag{1}$$

$$t = P + 2,\ s = P' + 1,\ y = P''.$$

The quantum numbers (P, P', and P''), which characterize the multiplet containing the ground state of the nucleus A (N, Z), have the values

$$(P, P', P'') = \begin{cases} (T_Z, 0, 0) & N, Z - \text{even} \\ (T_Z, 1, 0) & N, Z - \text{odd} \\ (T_Z, 1/2, \pm 1/2) & A - \text{odd} \end{cases}, \tag{2}$$

$a(A)$ and $b(A)$ are universal constants of the isobaric nuclei. The Wigner formula corresponds to the energy of the system in which the Casimir operator C_2 of the SU(4) group was used. The feasibility of using two other Casimir operators, C_3 and C_4, of the SU(4) group, in addition to C_2, was mentioned in Ref. 7. Until now, however, the contribution of these terms has not been detected.[8]

We have studied the mass differences of the nuclei $A \geqslant 216$ in order to determine the contribution of higher-order Casimir operators to the mass formula. The region $A \geqslant 216$ corresponds to the region in which the SU(4) symmetry has been restored and to the region of highly precise experimental studies of masses. The mass formula was given in SU(4) in the invariant form with allowance for violations due to the Coulomb and pairing interactions

$$M(A, T_Z) = a(A) + b(A)\frac{1}{2}(t^2 + s^2 + y^2) + c(A) t s y + d(A)(t^2 s^2 + t^2 y^2$$

$$+ s^2 y^2) + e(A)(t^4 + s^4 + y^4) + V_C(A, Z) + \delta M_p(A), \qquad (3)$$

where $a(A)$, $b(A)$, $c(A)$, $d(A)$, and $e(A)$ are invariant parameters which are universal for the isobaric nuclei. The difference in the Coulomb energies ΔV_C was determined by a special analysis of the data on the location of analogous resonances of nuclei for $A \geqslant 60$ (Ref. 9):

$$\Delta V_C = V_C(A, Z+1) - V_C(A, Z)$$

$$= (2Z + 1) A^{-1/3} (703(1 - 1.28 A^{-2/3}) \pm 4.5) \text{ keV} \qquad (4)$$

$\delta M_p(A)$ is the known phenomenological pairing correction corresponding to $\pm\Delta(A)$ for even nuclei and to zero for odd nuclei. A study of the mass difference of isobaric nuclei makes it possible to determine the parameters

$$b(A, T_0) = b(A) + e(A) T_0 (T_0 + 1); c(A);\ d_{\text{eff}}(A) = d(A) + \Delta(A)/T_0(T_0 + 3); e(A) \qquad (5)$$

($T_0 = 26$ is the average isospin in the investigated region) and hence to refine the mass formula for heavy nuclei. We have used a total of 125 values of the mass differences, taken from the data of Wapstra and Bos,[10] of which 30 highly accurate values were used to calculate the errors. All these data gave 24 independent sets of parameters for the odd nuclei and 28 sets for the even nuclei. The results, illustrated in Figs. 1a–1d, allow us to make the following assumptions.

1. The mass differences of the isobaric nuclei are described by Eq. (3) in the restoration scheme of the SU(4) symmetry with an accuracy of ~200 keV in the region $A \geqslant 216$.

2. The parameter $b(A)$ describes the contribution of the effective two-particle spin-isospin interaction, which has the structure

$$\sum_{i \neq j = 1}^{A} [(\vec{\sigma}^i \vec{\sigma}^j) + (\vec{\tau}^i \vec{\tau}^j) + (\vec{\sigma}^i \vec{\sigma}^j)(\vec{\tau}^i \vec{\tau}^j)]. \qquad (6)$$

The parameter $b(A, T_0)$ is described by an approximate empirical expression

$$b(A, T_0) = (-3(A - 230) + 738) \pm 5 \text{ keV}. \qquad (7)$$

3. The parameter $c(A)$, which describes the contribution to the odd nuclei of the effective three-particle spin-isospin interaction of the structure

$$\sum_{i \neq j \neq k = 1}^{A} (\epsilon_{\alpha\beta\gamma} \sigma_\alpha^i \sigma_\beta^j \sigma_\gamma^k + \epsilon_{\alpha\beta\gamma} \tau_\alpha^i \tau_\beta^j \tau_\gamma^k + \epsilon_{\alpha\beta\gamma} \epsilon_{\mu\nu\chi} \sigma_\alpha^i \sigma_\beta^j \sigma_\gamma^k \tau_\mu^i \tau_\nu^j \tau_\chi^k) \qquad (8)$$

is roughly described by the average value

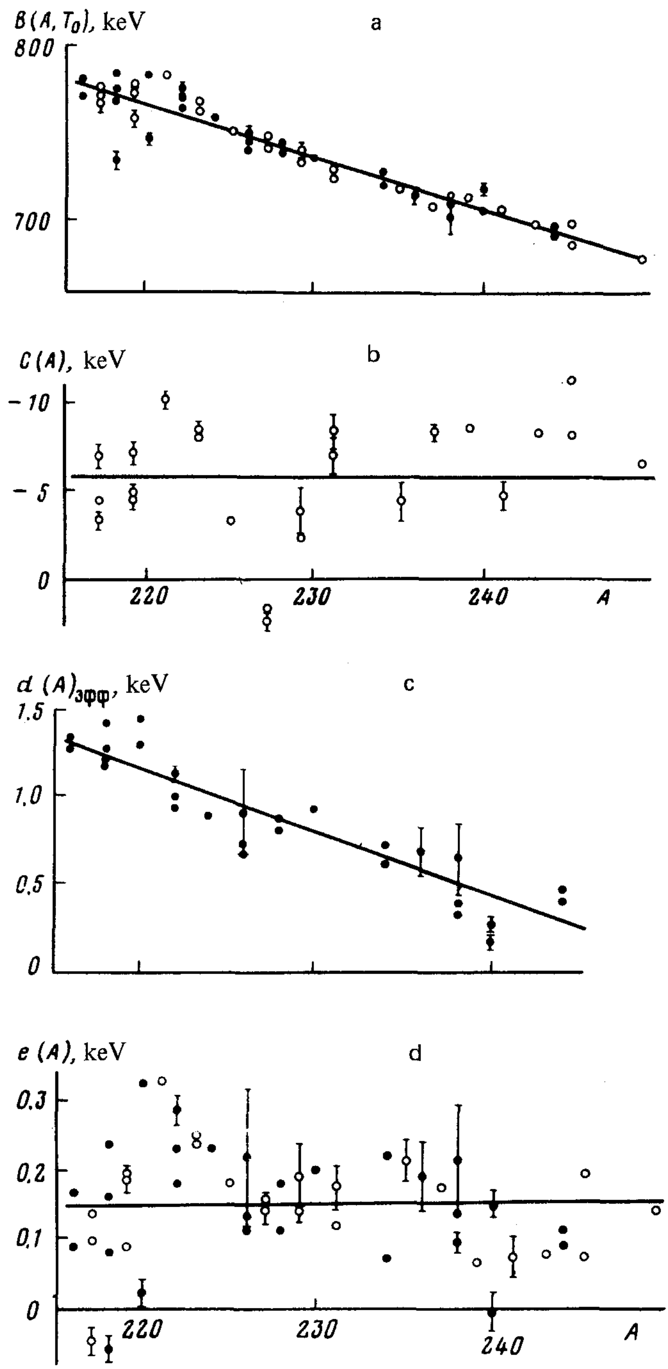

FIG. 1. Experimental values of the parameters of the mass formula (3) $b(A, T_0)$, $c(A)$, $d_{\rm eff}(A)$, and $e(A)$, respectively, and their approximation (solid lines) for the even nuclei (•) and the odd nuclei (○). The absence of an error at the experimental point shows that the experimental error is smaller than the minimum scale of the plots.

$$c(A) = -6.0 \pm 2.0 \text{ keV}. \quad (9)$$

The total contribution of this type to the mass differences of odd nuclei is ~200 keV.

4. The parameter $d_{eff}(A)$, which describes the average combined pairing contribution of the isobar, $\Delta(A)$, and of the effective four-particle spin-isospin forces, $d(A)$, to the even nuclei (the individual contributions cannot be determined), is approximated by the expression

$$d_{eff}(A) = (0.81 - 0.4(A - 230)) \pm 0.09 \text{ keV}. \quad (10)$$

The total contribution of this type to the mass differences of even nuclei is ~600 keV.

5. The parameter $e(A)$, which describes the contribution of the effective four-particle spin-isospin interaction, is roughly approximated by the average value

$$e(A) = 0.15 \pm 0.05 \text{ keV}. \quad (11)$$

However, its values are sensitive to the undetermined differences of the Coulomb energies (4). The predicted contribution of this term is greater than $b(A, T_0) \sim 200$ keV.

6. The total effect of the spin-orbit mechanism of symmetry violation in a charged $p\bar{n}$-excitation channel in the investigated region is

on the parameter $b(A, T_0) \gtrsim 0.5\%$, on $d_{eff}(A) \gtrsim 10\%$, on $c(A) \gtrsim 30\%$, and

on $e(A) \gtrsim 50\%$.

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