Large-angle elastic scattering

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A power law for the decrease of a large-angle scattering cross section, which is a consequence of the analytic properties of the transferred-momentum amplitude, has been obtained within the formation based on the solution of a simultaneous equation for the amplitude in the quantum field theory.

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Experimental studies of large-angle scattering have led to the discovery of a power law for the decrease of cross sections in this kinematic region. A detection of these systematic features was very important in the understanding of the structure of hadrons.[1] They showed that the effective inhomogeneity of close-range interaction or the point components of hadrons are crucial in large-angle scattering.

It is known that a scattering in the region of small values of t/s can be described in terms of dynamic ratios of the quantum field theory. In this letter we show that the power law for the decrease of cross sections, which is a consequence of the analytic properties of the transferred-momentum amplitude, is valid for a scattering amplitude which satisfies a simultaneous dynamic equation (a relativistic generalization of the main equation of the quantum theory of attenuation) in the region of fixed angles (t/s is fixed).

We shall carry out a comparison with the experimental data and discuss the role of forward and backward interaction radii in the scattering at angles close to 0° and 180°.

The simultaneous equation for the scattering amplitude in the operator notations has the form[3]

$$F = U + iUDF. \tag{1}$$

We shall represent the kernel of the integral equation—the generalized reaction matrix $U(s, t)$—in the form

$$U(s, t) = U_1(s, t) + U_2(s, u). \tag{2}$$

Here the $U_1(s, t)$ function is determined by the dynamic properties of the direct process, and $U_2(s, u)$ is determined by the properties of the exchange process. The scattering amplitude can be written in the form

$$F(s, t) = F_1(s, t) + F_2(s, u),$$

where the $F_1(s, t)$ and $F_2(s, u)$ functions are determined by the integral equations

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$$F_1 = U_1 + iU_1DF_1 + iU_2DF_2,$$

$$F_2 = U_2 + iU_1DF_2 + iU_2DF_1. \tag{3}$$

The solution of Eqs. (3) in the impact-parameter representation with allowance for the relative suppression of the exchange process has the form

$$F_1(s,t) = \frac{s}{2\pi^2}\int_0^\infty d\beta \frac{u_1(s,\beta)}{1-iu_1(s,\beta)} J_0(\sqrt{-\beta t}),$$

$$F_2(s,u) = \frac{s}{2\pi^2}\int_0^\infty d\beta \frac{u_2(s,\beta)}{[1-iu_1(s,\beta)]} J_0(\sqrt{-\beta u}), \tag{4}$$

where $\beta = b^2$.

The analytic properties of the $F_{1(2)}[s, t(u)]$ functions make it possible to write the dispersion relations in t and u variables, which, together with Eqs. (3), give the following representations for the $u_{1(2)}(s,\beta)$ functions:

$$u_{1(2)}(s,\beta) = \int_{\mu^2_{1(2)}}^\infty \rho_{1(2)}(s,x) K_0(\sqrt{x\beta})\, dx. \tag{5}$$

It follows from this expression that the $u_{1(2)}(s,\beta)$ functions have a singularity at the point $\beta = 0$ (the cut $\beta \epsilon [0, -\infty)$. This singularity gives rise to the power law for decrease of the cross section in the large-angle scattering.[3]

Let us examine the expression

$$u_{1(2)}(s,\beta) = ig_{1(2)}(s)\exp(-\mu_{1(2)}\sqrt{\beta}), \tag{6}$$

which takes into account in a straightforward manner the analytic properties that follow from the representation (5). An increase of the total interaction cross sections requires that the $g_1(s)$ function increase as $s \to \infty$. Taking into account the polynomial limitation of the U matrix, we set $g_{1(2)}(s) \sim s^{\lambda}$ 1(2)/2, where $\lambda_2 \leqslant \lambda_1$. The last condition ensures a power-law decrease of the backward scattering cross section, which was observed experimentally.

To calculate the integrals (4), we must switch from integration along the positive semiaxis $\beta \epsilon [0, \infty]$ to integration along the contour which includes this semiaxis and which closes on the large circle in the complex β plane. In calculating the integrals of the $f_{1(2)}(s,\beta)$ functions which have a singularity at $\beta = 0$, it is convenient to switch to the functions $f_{1(2)}(s, \beta+\beta_0)$, $\beta_0 > 0$, thereby shifting the origin of the cut to the point $\beta = -\beta_0$, and then to switch to the limit $\beta_0 \to 0$ in the obtained expressions. The contour in this case bypasses the cut $\beta \epsilon [-\beta_0, -\infty]$, and the calculated values of the integrals converge uniformly on the expressions (4), which determine the amplitudes of $F_{1(2)}[s, t(u)]$.

In the scattering at angles close to 90°, the main contribution, as already mentioned, gives a singularity at the point $\beta = 0$. Calculating the contributions to

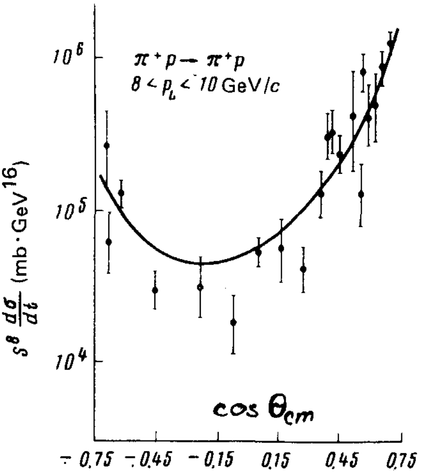

FIG. 1.

$F_1(s, t)$ and $F_2(s, u)$ from the corresponding cuts, we obtain the following expression for the large-angle differential scattering cross section:

$$\frac{d\sigma}{dt} \approx \left(\frac{1}{s}\right)^{\lambda_1 + 3} \left\{ (1 - \cos\theta)^{-3/2} + \left[\frac{g_2(s)}{\mu_1 g_1(s)} (\mu_2 - 2\mu_1) + O\left(\frac{g_2(s)}{g_1^2(s)}\right) \right] \times (1 + \cos\theta)^{-3/2} \right\}^2 . \qquad (7)$$

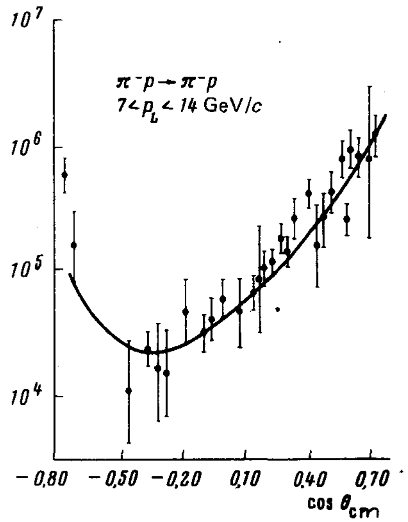

FIG. 2.

We can easily see that the obtained expression has the form $d\sigma/dt \sim s^{-N} f(\cos\theta)$, if the s dependence for the $g_1(s)$ function is the same as that for the $g_2(s)$ function. Figures 1 and 2 compare the angular dependence (7) with the data for $\pi^{\pm}p$ scattering. The general normalization and the coefficient in the square brackets were assumed to be free parameters. The expression (7) was obtained for the case in which the U matrix has the form (6).

A selection for the $u_{1(2)}(s, \beta)$ functions of other, more complex expressions than (6), for example, $u_{1(2)}(s, \beta) = i g_{1(2)}(s, \beta)(\mu_{1(2)}^2 \beta)^{-\gamma_{1(2)}} \ln^{\alpha_{1(2)}}(\mu_{1(2)}^2)$ $\times \exp(-\mu_{1(2)}\sqrt{\beta})$, does not change the main results, but leads to the appearance of additional factors that contain $\ln|t|$.[3] The expression for the cross section has the form

$$\frac{d\sigma}{dt} \sim \frac{1}{g_1^2(s)} \left[\frac{1}{(1+\gamma_1)^2} \left(\frac{\mu_1^2}{|t|} \right)^{1+\gamma_1} \frac{1}{\ln^{a_1} |t|/\mu_1^2} \phi_1(\ln^{-1}|t|/\mu_1^2) \right.$$

$$+ (-1)^{a_1 - a_2} \frac{g_2(s)}{g_1(s)} \left(\frac{\mu_1^2}{\mu_2^2} \right)^{2\gamma_1 + 1} \frac{1}{(1 + 2\gamma_1 - \gamma_2)^2} \left(\frac{\mu_2^2}{|u|} \right)^{1 + 2\gamma_1 - \gamma_2}$$

$$\left. \times \frac{1}{\ln^{2a_1 - a_2} |u|/\mu_2^2} \phi_2\left(\ln^{-1}|u|/\mu_2^2\right) \right]^2, \qquad (8)$$

$$\phi_i(0) = 1.$$

The expression (8), which was obtained by us from the analytic properties of the amplitude and from general considerations of the U matrix, coincides with that obtained within the context of the perturbation theory in *QCD*.[4]

The behavior of the cross sections in the region of small values of t and u is controlled by the effective forward and backward interaction radii[5] $R_{1(2)}(s) = \mu_{1(2)}^{-1} \ln g_{1(2)}(s)$. In fact,

$$\left.\frac{d\sigma}{dt}\right|_{t=0} \sim R_1^4(s), \quad \left.\frac{d\sigma}{du}\right|_{u=0} \sim g_2^2(s) g_1^{-4}(s) R_2^4(s),$$

and the scattering amplitude for the angles close to 180° contains an additional factor $\exp\{-\mu_2 [R_1(s) - R_2(s)]\}$, which has an obvious geometric meaning.

Conversely, the relative contribution of the direct and exchange interactions in the region of fixed scattering angles, in which the impact parameters $b \sim 0$ play a part, is determined by the ratio of the corresponding intensities: $g_2(s)/g_1(s)$.

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