Thermal fluctuations of a pion field near the π-condensate critical point

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The important role of thermal fluctuations of the pion field near the critical point of pion condensation is demonstrated. These fluctuations radically change the phase transition and the thermodynamic properties of the system.

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The pion condensation, one of the more interesting phenomena whose existence has been predicted in nuclear matter at sufficiently large density,[1] evidently has a larger density than that of ordinary nuclei $n_0 = 0.5$ (below we shall use the pion units $\hbar = c = m_\pi = 1$). The only way compressed nuclear matter can be obtained under laboratory conditions is by collision of high-energy heavy ions ($\sim$1 GeV/nucleon). The mechanisms of such reactions so far have not been studied in depth. The available experimental data[2] do not give direct information on the degree of compression, but indicate that the produced hadronic matter is heated strongly to temperatures T= 50–100 MeV, depending on the collision energy. Can a π-condensate phase transition manifest itself under such extreme conditions? The solution of this problem is important for setting up experiments with heavy pions and analyzing their results. The properties of pion condensation at finite temperatures are of great interest in the study of phase-transition-related dynamic effects in neutron stars.[3]

There are now available several studies (see, for example, Refs. 4–8) in which the critical density and energy gain of a π-condensed phase transition at $T \neq 0$ were calculated in the average-field approximation. Our goal in this paper is to show that strong pion-field fluctuations, which make a classical analysis impracticable, occur at finite temperatures.

In our approach the key role is played by the delayed Green's function of a pion in a thermodynamically stable system $D_R(\omega, k; n, T) = \omega^2 - 1 - k^2 - \Pi(\omega, k; n, T)$, where Π is the polarization operator which depends on the pion 4-momentum (ω, k), as well as on the density n and temperature T of hadronic matter (the arguments of n and T, as a rule, are dropped). According to Ref. 1, for $\omega < kv_F$ and $k \sim k_0$ (k_0 is the wave number of the condensate field, and v_F is the Fermi velocity) $D_R(\omega, k)$ can be represented in the form

$$D_R^{-1}(\omega, k) = (1 - \alpha)\omega^2 + i\beta\omega - \omega^2(k), \tag{1}$$

$$\omega^2(k) = \omega_0^2 + \frac{\gamma}{4k_0^2}(k^2 - k_0^2)^2. \tag{2}$$

The parameters α, β, γ, and k_0, which depend on n and T, can be expressed in terms of the polarization operator of a pion in the medium.[1] We need only to know that they are of the order of unity in pion units and that near the critical point

$$\omega_0^2 = \kappa(n_c(T) - n), \quad \kappa \sim 1, \tag{3}$$

where $n_c(T)$, the critical density at the start of pion condensation, expressed as a function of temperature, was calculated while ignoring the fluctuations.

It is well known that as the critical point is approached ($\omega_0^2 \to 0$), the fluctuations of the order parameter begin to increase in the presence of the soft mode. In our case this involves the isovector pion field $\vec{\phi}(\mathbf{r}, \tau)$ ($\phi_{\pi^\pm} = \phi_1 \pm i\phi_2/\sqrt{2}$, $\phi_{\pi^0} = \phi_3$). The Gibbs average of the product of two pion fields at different points (correlation function)

$$N_{ik}(r) = \langle \phi_i(\mathbf{r}_1, \tau_1)\,\phi_k(\mathbf{r}_2, \tau_2) \rangle |_{\tau_1 = \tau_2 - 0}$$

is expressed in terms of the Matsubari Green's function of a pion, which is related to $D_R(\omega, k)$ by simple relations.[9] Since the conditions in an isotopically symmetric homogeneous system are the same for all components of the pion field, $N_{ik} = N\delta_{ik}$ and it depends solely on $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Using standard procedures, we can obtain the following expression for the correlation function:

$$N(r) = -\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{kr}} \int_{0+}^{\infty} \frac{d\omega}{2\pi} \cot\frac{\omega}{2T}\, 2\,\mathrm{Im}\, D_R(\omega, k). \tag{4}$$

The expression (4) includes thermal as well as quantum fluctuations which do not vanish at $T = 0$. Below we shall not analyze the quantum fluctuations (see Ref. 10), which is equivalent to substituting $n(\omega)$ for $1/2 \coth \omega/2T$, where $n(\omega) = [\exp(\omega/T) - 1]^{-1}$ is the Bose distribution function.

Using the parametrization (1) and keeping only the main terms as $\omega_0^2 \to 0$, we finally find for the correlation function of thermal fluctuations

$$N(r) = 4 \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{kr}} \int_{0+}^{\infty} \frac{d\omega}{2\pi} \frac{n(\omega)\,\beta\omega}{[\omega^4(k) + \beta^2\omega^2]} . \tag{5}$$

The frequency integral in (5), which is determined by a single parameter $\xi = \beta T/\omega^2(k)$, can be easily calculated in two limiting cases: $\xi \ll 1$ and $\xi \gg 1$.

Integration over the momenta in Eq. (5) can be easily carried out if we assume that the integral is concentrated in the region $\Delta k \sim \omega_0$ near $k = k_0$. As a result, we obtain

$$N(r) = N(0) \frac{\sin k_0 r}{k_0 r} \exp\left[-\frac{r}{r_c}\right], \tag{6}$$

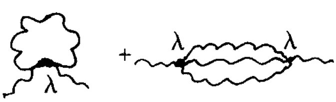
where

$$N(0) = \begin{cases} \dfrac{k_0^2}{2\pi\sqrt{\gamma}} \dfrac{T}{\omega_0}, & T >> \omega_0^2 \qquad (7a) \\[2ex] \dfrac{k_0^2 \beta}{12\sqrt{\gamma}} \dfrac{T^2}{\omega_0^3}, & T << \omega_0^2 \qquad (7b) \end{cases} .$$

This function is characterized by two lengths: it oscillates rapidly at distances $\sim 1/K_0$ and decreases smoothly at the length $r_c = \sqrt{\gamma/\omega_0^2}$, which is equivalent to the correlation radius. As the critical point is approached ($\omega_0^2 \to 0$) $r_c \to \infty$ and, as usually happens in the case of second-order phase transitions, the system develops a long-range interaction.

We shall determine the time at which the fluctuations in the pion polarization operator reach a significant level as the critical point is approached. We shall analyze the following diagram:

$\Delta\Pi$ =



(8)

In this diagram the point represents the amplitude of the local interaction of pion quasiparticles $H' = (\lambda/4)\phi^4$. In general, λ is a complex function of the kinematic variables and of the parameters of the medium.[1] For crude estimates, we assume that λ is a constant. By making such assumption, we can easily calculate the diagrams (8). Specifically, the first diagram is equal to $4\lambda N(0)$.

The effect of the second diagram (8), which includes additional powers of T at low temperatures, is small compared with that of the first diagram at high temperatures if the parameter $\lambda T/\omega_0^2$ is small (see below). For qualitative estimates, we shall therefore limit ourselves to the analysis of the first diagram (8). Since $\Delta\Pi$ is independent of ω and k, we can compare it with the magnitude of the "gap" in the pion spectrum ω_0^2. Therefore, it follows from the expressions (8a) and (8b) that for a constant $\omega_0^2 < 1$ the effect of fluctuations is small ($\Delta\Pi \ll \omega_0^2$) only at sufficiently low temperatures $T \ll T_f$, where

$$T_f = \begin{cases} \left(\dfrac{3\sqrt{\gamma}}{k_o^2\,\beta\,\lambda}\,\omega_o^5 \right)^{1/2}, & T_f << \omega_o^2 \quad (9a) \\[2ex] \dfrac{\pi\sqrt{\gamma}}{2k_o^2\,\lambda}\,\omega_o^3, & T_f >> \omega_o^2 \quad (9b) \end{cases}$$

For $\lambda \gtrsim 1$ only the upper part of this formula is in effect, and the region in which (9b) can be used appears at sufficiently small λ. The expressions (8) and (9), which were obtained by assuming that $\omega_0^2 > 0$, remain valid below the critical point ($\omega_0^2 < 0$), where $c|\omega_0^2|$ must be substituted for ω_0^2 ($1 < c \leqslant 2$ depends on the spatial structure of the condensate field).[1]

A classical calculation, in which the fluctuations are ignored, gives $T_c \sim \sqrt{|\omega_0^2|}$ (Ref. 5) and, as can easily be seen from Eqs. (9), $T_c \gg T_f$ when $|\omega_0^2| < 1$ and the λ's are not too small. Thus the thermal fluctuations of the pion field, which change dramatically the entire picture of the phase transition and the thermodynamics of the system, acquire a crucial importance long before the critical point is approached. In this respect the pion condensation differs drastically from the other phase transitions initiated at $k=0$, in which the fluctuations are large only in a narrow region near the critical point.

To illustrate the importance of fluctuations in the thermodynamic characteristics of the system, we shall estimate their contribution to the heat capacity ΔC at low temperatures. Using the expression for the free energy in terms of the Green's excitation function[11] and performing calculations analogous to those for N, we obtain for $T \ll \omega_0^2$

$$\Delta C = \frac{\beta k_o^2}{2\sqrt{\gamma}}\,\frac{T}{\omega_o} - \frac{3\,\pi^2 \beta^3 k_o^2}{20\,\sqrt{\gamma}}\,\frac{T^3}{\omega_o^5} + \cdots .$$

It follows from this that the heat capacity and hence the entropy of the system increase sharply with decreasing ω_0^2. In the heavy-ion collisions this takes the form of a slowing down or even cessation of the rise of temperature with the collision energy in a certain energy range.

We conclude from the given analysis that the critical fluctuations of the pion field are the clue that should be used to search for a π-condensate phase transition in heavy-ion collisions.

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