Nonlinear saturation of SMBS in a rarefied nonisothermal plasma

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Nonlinear saturation of SMBS in a nonisothermal plasma due to generation of higher-order harmonics of sound is analyzed. The constraints imposed on the coefficient of nonlinear reflection in a rarefied plasma are determined.

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The theory of stimulated Mandel'shtam-Brillouin scattering (SMBS) as applied to laser plasma has a special relevance because of the constraints this process imposes on the heating efficiency.[1-3] Vinogradov *et al.*[4] showed, on the other hand, that the nonlinear effects, in particular, the nonlinear propagation of ionic sound in nonisothermal plasma, must be taken into account in order to calculate correctly the coefficient of reflection due to SMBS from rarefied layers of plasma. This nonlinearity was incorporated into the theory of SMBS in a plasma by Gorbunov *et al.*[5] The results of their study, however, did not change the theoretical understanding of strong scattering. In this letter we show that the generation of harmonics analyzed by Gorbunov *et al.*[5] sharply restricts the SMBS in a rarefied plasma.[1)]

The original equations for the electric-field amplitudes of the incident electromagnetic wave E_0 and backward scattered wave E_1, and for the amplitudes of the fundamental harmonic ($n_1 = \delta n_1/n_e$) and second harmonic ($n_2 = \delta n_2/n_e$) of sound waves have the form

$$\frac{dE_0}{dx} = -\frac{1}{2}\alpha k E_1 n_1 \exp(i\int^x \Delta k dx); \qquad \frac{dE_1}{dx} = -\frac{1}{2}\alpha k E_0 n_1^* \exp(-i\int^x \Delta k dx)\cdot$$

$$\frac{dn_1}{dx} = k\frac{E_0 E_1^*}{16\pi n_c \kappa T_e}\exp(-i\int^x \Delta k dx) + k n_1^* n_2; \qquad \frac{dn_2}{dx} = -2kn_1^2, \qquad (1)$$

where x is the coordinate in the direction of propagation of the pump wave, $k = \omega_0/c(1 - n_e/n_c)^{1/2}$ (ω_0 is its wave number and frequency), $n_e(x)$ is the electron density, n_c is the critical density, $\alpha = n_e/(n_c - n_e)$, $\Delta k(x)$ is the frequency difference of the wave numbers of three interacting waves, and T_e is the temperature plasma electrons.

First, we shall show that in a simple model (see Ref. 7) of a homogeneous plasma layer of thickness l the flow of energy of one sound wave into the second harmonic sharply reduces the intensity of SMBS.

Let us analyze the set of equations (1) for $\Delta k = 0$ and for the boundary conditions (see Ref. 7)

$$E_0(0) = (8\pi q_0/c)^{1/2}(1 - n_e/n_c)^{-1/4}, \quad n_1(0) = n_2(0) = 0; \quad E_1(l) = 0, \tag{2}$$

where q_0 is the density of the energy flux of a pump wave in a vacuum. The thickness of the layer is limited by the condition $kl < (m_i/m_e)^{1/2}$ if the sound dissipation is ignored.

The set of equations (1) has first integrals

$$|E_0|^2 - |E_1|^2 = C_1, \quad \frac{|E_1|^2}{8\pi n_c \kappa T_e} + a\left(|n_1|^2 + \frac{1}{2}|n_2|^2\right) = C_2;$$

$$\mathrm{Im}\left(\frac{E_0 E_1^*}{8\pi n_c \kappa T_e} n_1^* - n_1^2 n_2^*\right) = 0 \qquad (\text{for } \Delta k = 0). \tag{3}$$

After incorporating Eq. (2), the reflection coefficient $R = |E_1(0)/E_0(0)|$ is related to C_1 by the relation $C_1 = E_0^2(0)(1 - R^2)$.

Introducing the amplitudes and phases of the waves $E_i = |E_i| \exp i\Phi_i$, $n_m = |n_m| \exp i\psi_m$ and taking into account Eqs. (2) and (3), we reduce Eqs. (1) to the following form:

$$\frac{dz}{d\xi} = -\frac{1}{2}\sqrt{fF}, \quad \frac{dy}{d\xi} = -\frac{R\sqrt{2}}{\alpha} f \cos\chi, \quad \frac{d\chi}{d\xi} = \frac{R\sqrt{2}}{\alpha y}(f - y^2)\sin\chi, \tag{4}$$

where $\xi = kxE_0(0)\sqrt{\alpha/8\pi n_c \kappa T_e}$; $|E_0(x)|^2 = E_0^2(0)\,[1 - R^2(1 - z^2)]$; $\chi = \psi_2 - 2\psi_1$; $|n_2| = yRE_0(0)/\sqrt{4\alpha\pi n_c \kappa T_e}$; $f(y,z) = 1 - z^2 - y^2$; $F(y,z,\chi,R) = 1 - R^2(1 - z^2) - 2R^2/\alpha^2\, y^2/z^2\, f(y,z)\sin^2\chi$. Further, writing $p = (kl/\pi) \times E_0(0)\sqrt{\alpha/8\pi n_c \kappa T_e}$, we derive from Eq. (4) the following relation that determines $R(p)$:

$$\pi p/2 = \int_0^1 dz\, F^{-1/2}(y(z), z, \chi(z), R). \tag{5}$$

In the standard three-wave theory ($y = 0$) the right side of Eq. (5) is equal to $K(R)$, where K is an elliptic integral. From this follows the well-known conclusion (see Ref. 7) that reflection occurs when the threshold is exceeded $p = 1$, and $R^2 \approx 4(p - 1)$ near the threshold.

Let us analyze the corollaries of our equations (4) in the near-threshold region $R \ll 1$, assuming that $1 - z^2 \gg y^2$, which means that the amplitude of the second harmonic is small compared with the main perturbation. We can see that $y \sin\chi = 0$, which allows us to set $\chi = \pi$. Solution of the equation for y is found in explicit form $y(z) = (R\sqrt{2}/\alpha)(\arccos z - z\sqrt{1 - z^2})$, which, according to (5), gives

$$R^2 = \frac{4\alpha^2}{\alpha^2 + 2}(p - 1). \tag{6}$$

Since $\alpha \ll 1$ in a rarefied plasma, the reflection coefficient decreases sharply compared with the prediction of the three-wave theory. Physically, this means that generation of the second harmonic of sound is a much more efficient method of saturating SMBS than that involving the reduction of the strength of the pump wave. When

$$R \sim 2a/\pi \tag{7}$$

the amplitudes of the second harmonic and higher harmonics of sound are equal to the main amplitude. The value in (7) should be regarded as a limiting value, because the onset of effective turbulent dissipation of sound is expected to occur as a result of inversion of the sonic wave front.

A suppression of SMBS occurs even in an inhomogeneous plasma in which, in contrast with an absolute instability that leads to (6), there is a convective amplification.[1]

Ignoring the reduction of pumping, assuming that $\Delta k = \alpha kx/L$ is a linear function of the coordinate (here $x = 0$ is the point at which the phases of the waves are synchronized, and L is the scale of the nonuniform plasma density at this point), and assuming that $E_1(x) = E_1 \exp(-i/2 \int^x \Delta k dx + i/2 \int^x q dx)$; $n_m(x) = n_m \exp(-im/2 \int^x \Delta k dx - im/2 \int^x q dx)$, we derive from (1) the eikonal equation,

$$\frac{q^2(x)}{k^2} = a^2 \frac{x^2}{L^2} + a \frac{E_0^2(0)}{8\pi n_c \kappa T_e} - |n_1|^2 \frac{akx - qL}{akx + qL} . \tag{8}$$

The amplification of waves occurs only when two points in a complex plane x at which the waves stop $x_t [q(x_t) = 0]$ have the same real parts. From (8) we find $x_t = \pm i L/\alpha\ [\alpha E_0^2(0)/8\pi n_c \kappa T_e - |n_1|^2]^{1/2}$. It follows that SMBS stops when the intensity of sonic perturbations reaches the level $|n_1|^2 \approx \alpha E_0^2(0)/8\pi n_c \kappa T_e$. Incorporating the integral C_2 in (3), we find $\max|E_1|^2 \lesssim \alpha^2 E_0^2(0)$. This estimate corresponds to the constraint (7) on the reflection coefficient determined above.

Since the limiting value of the local reflection coefficient is known, we can estimate the total reduction of high-power radiation due to SMBS during its propagation in a rarefied plasma. It follows from (8) (see also Ref. 8) that the effective length of resonance interaction of waves is $\Delta x \sim L E_0(0)/\sqrt{8\alpha\pi n_c \kappa T_e}$. Thus we can write the following equation for the energy flux q_0 of a pump wave:

$$\frac{dq_0}{dx} \approx \frac{\Delta q_0}{\Delta x} \approx -\frac{R^2}{L}(a q_0 c n_c \kappa T_e)^{1/2} \lesssim -\frac{a^{5/2}}{L}(q_0 c n_c \kappa T_e)^{1/2} .$$

We see from this equation that a reflected energy flux $q_{\rm refl} \lesssim 2/5\ \alpha_{\max}^{5/2}\ (q_0 c n_c \kappa T_e)^{1/2}$, which comprises a small fraction of the incident energy flux, occurs when $n_{e\,\max} \ll n_c$ as a result of radiation propagation in the plasma from the vacuum to the density $n_{e\,\max}$, irrespective of the scale of the density nonuniformity.

1) The attenuation of sound waves analyzed in Ref. 6 for SMBS saturation is much less efficient than the harmonic generation.

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