Helical-axis stellarator with helical windings

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The equilibrium and stability of the plasma in a stellarator with a helical geometric axis and with helical external fields are analyzed.

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Stellarators have recently been attracting increasing interest as steady-state, plasma-confinement systems with rather high values of $\beta = 2\bar{p}/\mathbf{B}^2$. Estimates of the maximum value of β ($\beta_{max} \sim 5$–10%) for an "ordinary" stellarator, with a circular geometric axis, still leave some doubt that a successful fusion reactor can be designed around the stellarator concept. Analysis of the ordinary stellarator shows that β_{max} could be raised if it were possible to increase the rotational transform per unit length of the system without decreasing the depth of the magnetic well. This can in fact be done by twisting the geometric axis of the ordinary stellarator (by a reasonable amount). With the goal of maximizing β, therefore, it would be natural to consider the possibility of a composite stellarator: one with a three-dimensional axis and helical windings.

The ordinary stellarator with the optimum characteristics, which have thus far been determined only approximately, is definitely a three-dimensional system and thus difficult to describe. Although recently developed methods, which use expansions in small parameters,[1,2] permit analytic study of the equilibrium and stability, the expansion in Δ/a, which was used in Refs. 1 and 2, seriously restricts the applicability of the results (a is the average radius of the magnetic surface, and Δ is the displacement of the center of this surface with respect to the geometric axis). The condition $|\Delta|/a \ll 1$ is a natural condition in the method of inversion of variables,[1] and this method can be used to describe the entire plasma column, even in a composite stellarator, if a correct transformation is made from the reference quasicylindrical coordinate system (r, ψ, ζ) to the plasma "flux" coordinate system with straightened lines of force (a, θ, ζ).

For a composite stellarator with a helical geometric axis, of constant curvature k and constant twisting κ, the transformation $(r, \psi, \zeta) \to (a, \theta, \zeta)$ can be written in the simple form

$$r = \rho + \delta(\rho, \alpha, \zeta), \qquad \psi = \alpha + \lambda(\rho, \alpha, \zeta) \tag{1}$$

where

$$\rho \cos\alpha = a \cos\theta_1 + \Delta(a), \qquad \rho \sin\alpha = a \sin\theta_1,$$
$$\theta_1 = \theta + \eta(a)\sin\theta. \tag{2}$$

Transformation (1) describes a transformation to the vacuum flux coordinate system (ρ, α, ζ), which is related to the plasma flux coordinate system by the relations in (2), which are two-dimensional (!). Assuming that δ/ρ, λ, $|\Delta'|$, and η are small, as in Ref. 1, we can determine δ and λ' from the equilibrium and magnetostatic equations, which take a simple form in the flux coordinate system.[3] We can then use the conventional approach[3] to solve the remaining two-dimensional equations.

The resulting equation for the displacement Δ is linear for stellarators with a small shear, in which μ_{st} may be written $\mu_{st} = \mu_0 + \mu_1 a^2$. This equation is

$$[(\mu_{st}\Delta)' a^3]' + \frac{1}{\mu_J}[\mu_J^2 a^3 \Delta']' = -\frac{2p' a^2 k R^2}{\mu B_0^2} + \frac{p' a^2 R V_0''(\phi)}{\mu}\Delta, \tag{3}$$

where μ_J is the rotational transform associated with the current, μ_{st} is the stellarator rotational transform, $\mu = \mu_{st} + \mu_J$ is the total rotational transform, $V_0''(\phi)$ is the vacuum part of the derivative $d^2V/d\phi^2$, V is the volume, ϕ is the longitudinal flux inside the magnetic surface $a = \text{const}$, and $2\pi R$ is the total length of the geometric axis. For a large-shear stellarator, on the other hand, the equation for the displacement is nonlinear, and it reduces to (3) only if $a > |\Delta|$. In deriving the equation for the displacement we did not use the condition $|\Delta|/a \ll 1$ of Refs. 1 and 2, which greatly restricts the applicability of the results. Equation (3) describes a broad class of configurations, since three independent possibilities for producing the rotational transform are taken into account: the longitudinal current, the helical external fields, and the twisting of the axis. The left side of (3) is the sum of three terms corresponding to these possibilities; for convenience, two of the terms have been combined into a common term with $\mu_{st} = \mu_\epsilon + \mu_\kappa$, where μ_ϵ is the part of the rotational transform which is generated by the helical external fields, and $\mu_\kappa = -\kappa R$ is that which results from the twisting of the axis. The term with $V_0''(\phi)$ in (3), which corresponds to the particular method used to maintain the equilibrium in stellarators with helical fields,[4,1] may be ignored for a stellarator with a small magnetic bump or with a magnetic well (and this is the case of particular interest).

The approach which has been taken to describe the equilibrium configurations is exceedingly convenient for analyzing the stability of local perturbation modes. Using the Connor-Hastie-Taylor method[8] (supplemented with Zakharov's interpretation of this method[6]), the model approach of Pogutse and Yurchenko,[7] and the known metric of the flux coordinate system, we can derive a condition for stability with respect to ballooning modes in the stellarators:

$$\frac{1}{2}s^2 + \frac{p'Ra}{\mu^2 B_0^2}\left[B_0^2 V_0''(\phi) + kR\Delta\frac{(\mu_{st}' a^3)'}{\mu a^3}\right] - \frac{s}{2}\left(kR^2\frac{2p'}{\mu^2 B_0^2}\right)^2 > 0, \quad (4)$$

where $s = -a\mu'/\mu$ is the magnetic shear. The last term in (4), which is a destabilizing term ($s > 0$) in the case of tokamaks, is a stabilizing term ($s < 0$) in the case of stellarators. In analyzing the stability of local modes in stellarators, therefore, we should use the more stringent Mercier criterion.[8] This criterion can also be expanded in the flux coordinate system in a simple manner:

$$\frac{1}{4}s^2 + \frac{p'Ra}{\mu^2 B_0^2}\left[B_0^2 V_0''(\phi) + kR\Delta\frac{(\mu_{st}' a^3)'}{\mu a^3}\right] > 0. \quad (5)$$

If the vacuum magnetic bump is canceled by an outward displacement of the plasma column[2] [the second term in brackets in (5)], a restriction on β arises from the equilibrium conditions. Estimates show that in this case β_{max} for a composite stellarator can be much larger, by a factor of $1 + \mu_\kappa/\mu_\epsilon$ (this is a factor of two or three), than for an ordinary stellarator (with a circular axis). The equilibrium and stability of the plasma in the composite stellarator are determined exclusively by the local aspect ratio, as can be seen from (3)-(5), so that there is no restriction on the total length of the system, in contrast with the case of the ordinary stellarator.

The method used in this letter can also be used to describe more complicated configurations: stellarators with a large shear, with severely distorted magnetic surfaces, or with a combination of helical harmonics. Such studies would be required to optimize the system and to generate a more accurate estimate of β_{max}.

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