Weinberg angle and the proton lifetime in asymptotically free and supersymmetric grand unification models

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The renormalized Weinberg angle, the mass of intermediate vector bosons of the weak interaction, and the proton lifetime are determined in asymptotically free and supersymmetric grand unification models.

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It has been established in recent years that low-energy physics is described by a standard model based on the gauge group $S^CU(3)\times SU(2)\times U(1)$, the strong interaction is described by the color group $S^CU(3)$, and the weak and electromagnetic interactions are described by the Glashow-Weinberg-Salam model $[SU(2)\times U(1)]$. The standard model breaks down spontaneously to the $S^CU(3)\times U_e(1)$ group at energies of the order of 100 GeV. The multiplet composition of the standard model, which has been determined primarily at currently available energies, reduces to three or four generations of chiral Fermi particles, and the spontaneous symmetry breaking of the standard model to the symmetry $S^CU(3)\times U_e(1)$ is apparently accomplished by one or several isodoublets[1,2] of the $S^CU(3)\times SU(2)$ group. The crucial task of the current stage of development of the elementary particle theory is the determination of the symmetry group and the multiplet composition of high-energy physics. The grand unification models partially solve this problem. In the minimal grand unification model $[SU(5)]$, the Georgi-Glashow model,[1] it is assumed that the multiplet composition of high-energy physics reduces to several generations of chiral Fermi particles and that the hierarchy of gauge interactions and spontaneous symmetry breaking is achieved due to Higgs particles (24-multiplet, 5-multiplet, and conceivably 45-multiplet). A more comprehensive physical picture of the interaction and a broader particle spectrum have been realized by asymptotically free grand unification models, which were first formulated by one of the authors (E.S.F.), in collaboation with Kalashnikov and Konshtein.[2-4] The most important advantages of asymptotically free models over the standard models can be summarized as follows.

1) These models, as well as the supersymmetric models, are intrinsically consistent. In contrast with the standard models and their problem of zero charge[5] associated with Higgs interaction, our models have asymptotic freedom for all types of interaction.

2) Since the model as a whole is asymptotically free, the perturbation theory applies to all types of interaction at high energies.

3) The requirement of asymptotic freedom for all interactions imposes certain constraints on the multiplet composition of the model and established certain correlations between different interaction constants and particle masses. This reduces the number of arbitrary parameters of the theory as compared with the ordinary grand unification models.

Our goal in this letter is to identify the most striking qualitative implications of asymptotically free as well as supersymmetric SU(5) grand unification models. We are referring to the characteristics already identified experimentally or potentially identifiable with modern experimental tools; specifically, the calculation of the renormalized Weinberg angle ($\sin^2\theta_W$), the mass of intermediate vector bosons of weak interaction $M_{W^\pm}, M_Z$, the unification points M_X of the coupling constants $\alpha_3, \alpha_2, \alpha_1$ of the gauge groups $S^CU(3)$, SU(2), and U(1), as well as the proton lifetime τ_p.

1. Basic Formulas

The basic equations used for subsequent calculations are the renormalization-group equations for the effective gauge coupling constants $(\alpha_3, \alpha_2, \alpha_1)$ of the $S^CU(3)\times SU(2)\times U(1)$ symmetry, which is realized after spontaneous breaking of the original SU(5) symmetry In the one loop approximation we have

$$\alpha_n^{-1} = \alpha_0^{-1} + \frac{b_n}{4\pi}\ln\frac{K^2}{M_X^2}\,; \quad a = \frac{8}{3}b_3 - \frac{5}{3}b_1 - b_2\,, \tag{1}$$

$$\frac{\alpha}{\alpha_2} = \sin^2\theta_W = \frac{1}{a}\left[b_3 - b_2 + \frac{5}{3}\frac{\alpha}{\alpha_3}(b_2 - b_1)\right], \tag{2}$$

$$\ln\frac{M_X}{M_W} = \frac{2\pi}{a\alpha}\left(1 - \frac{8}{3}\frac{\alpha}{\alpha_3}\right), \tag{3}$$

$$\frac{\alpha}{\alpha_0} = \frac{1}{a}\left\{b_3 - \frac{\alpha}{\alpha_3}\left(\frac{5}{3}b_1 + b_2\right)\right\}, \tag{4}$$

$$\frac{\alpha}{\alpha_1} = \frac{3}{5}\cos^2\theta_W \quad ; \quad \alpha^{-1} = \alpha_2^{-1} + \frac{5}{3}\alpha_1^{-1} \;, \tag{5}$$

$$M_{W^\pm} = 37{,}3\,/\,\sin\theta_W\ \text{GeV}; \quad M_Z = 74{,}6/\,\sin 2\,\theta_W\ \text{GeV}, \tag{6}$$

$$\tau_p = \tau_p^o\left(\frac{M_X}{M_X^o}\right)^4 \left(\frac{\alpha_o^o}{\alpha_o}\right)^2 \left(\frac{A_3^o}{A_3}\right)\left(\frac{A_2^o}{A_2}\right)^2; \quad \begin{array}{l} A_3 = (\alpha_3\ (M_X^2)/\alpha_o\,)^{4/b_3} \\ A_2 = (\alpha_2(M_X^2)\ /\ \alpha_o)^{9/4\,b_2} \end{array} \tag{7}$$

where $\tau_p^0 = 1.4 \times 10^{31\pm2}$ yr is the proton lifetime in the minimal SU(5) model with three chiral generations and one Higgs 5-multiplet, in which

$$b_3 = 7;\ b_2 = 19/6;\ b_1 = -4{,}1;\ \alpha_o = 0.0242;\ A_3^o = 4{,}72;\ A_2^o = 1{,}37\ ;$$
$$M_X^o = 6.66 \cdot 10^{14}\,\text{GeV};\ \sin^2\theta_W = 0{,}207;\ M_{W\pm} = 82\ \text{GeV};\ M_Z = 92{,}08\ \text{GeV}.$$

The b_n coefficients of the SU(n) group, which depend on the spin (S) and on the model, are

$$b_n = \frac{.1}{6}\sum_S (1 - 12\,S^2)\,C_n\,, \tag{8}$$

where the sum is taken over the particle helicities ($S = \pm 1$ for the vector gauge field and $S = \pm ½$ for the spinor field), and the C_n coefficient is equal to ½ for the fundamental representation, to n for the adjoint representation, to $n/2 - 1$ for the antisymmetric representation, and to $n/2 - 1$ for the symmetric representation.

The normalized values of C_n pertain to real Bose fields and chiral (or Majorana) Fermi fields. The value of the C_n coefficient doubles for complex Bose particles and total Dirac Fermi particles. We can see from Eqs. (3) and (7) that small corrections for the right side of Eq. (3) can change substantially the proton lifetime τ_p. Thus the value of τ_p changes dramatically as a result of introduction of the two-loop corrections for the renormalization-group equations. We can show that these radiation corrections lead to the addition of the factor Δ to the basic formulas

$$\Delta(\sin^2\theta_W) = \frac{1}{\alpha}\left\{\frac{5}{3}(b_2 - b_1)\delta_3 + (b_3 - b_2)\delta_1 + (b_1 - b_3)\delta_2\right\}, \tag{2a}$$

$$\Delta\left(\ln\frac{M_X}{M_W}\right) = \frac{2\pi}{\alpha\,\alpha\,(M_X)}\left\{-8/3\,\delta_3 + 5/3\,\delta_1 + \delta_2\right\}, \tag{3a}$$

$$\Delta(\alpha/\alpha_o) = \frac{1}{\alpha}\,\{-(5/3\,b_1 + b_2)\delta_3 + b_3\,(\delta_2 + 5/3\,\delta_1\,)\}, \tag{4a}$$

$$\Delta(\alpha/\alpha_1) = \frac{1}{\alpha}\,\{(b_1 - b_2)\delta_3 + (b_2 - b_3)\delta_1 + (b_3 - b_1)\,\delta_2\}, \tag{5a}$$

$$\Delta\left(a/a_3\right) = \delta_3 \tag{9}$$

$$\delta_n = \sum_{i=1}^{3} \frac{\beta^{ni} a}{4\pi b_i} \ln \frac{a_i}{a_0}, \tag{10}$$

where (Ref. 7)

$$\beta^{ni} = -\begin{vmatrix} \frac{19}{15} n_f & \frac{3}{5} n_f & \frac{44}{15} n_f \\ \frac{1}{5} n_f & \frac{49}{3} n_f - \frac{136}{3} & 4 n_f \\ \frac{11}{20} n_f & \frac{3}{2} n_f & \frac{76}{3} n_f - 102 \end{vmatrix}. \tag{11}$$

Here n_f is the number of chiral generations of spinor particles in the model.

2. Asymptotically Free SU(5) Models

We shall calculate the relevant values only for the most realistic, asymptotically free SU(5) model, which was recently proposed by one of the authors (E.S.F.) and Konshtein.[4] This model, which represents the most complete generalization of the first, asymptotically free SU(5) model,[2] contains the following multiplets: (a) a set of 3-4 generations of chiral Fermi particles, (b) a set of Fermi and Higgs particles in the adjoint (24) representation, and (c) 5-multiplet and anti 5-multiplet of the total Fermi particles and complex Higgs particles.

Spontaneous SU(5)-symmetry breaking gives rise to the standard model with the following spectrum of light particles:

1) A set of 3–4 generations of chiral Fermi particles; 2) two sets of isodoublets (1,2) and (1,2) of Higgs particles; 3) a triplet of complete Fermi particles (3, 1); and 4) an isodoublet of complete Fermi particles (1,2).

The corresponding b_n coefficients for the three generations are

$$b_3 = 7 - 2/3 = 19/3;\ b_2 = 10/3 - 1/3 - 2/3 = 7/3;$$
$$b_1 = -4 - 2/3 \times 2/5 - 3/5 = -73/15.$$

After incorporating the 2-multiplet corrections, we find for the relevant quantities: $\sin^2\theta_W = 0.2108$, $\tau_p = 2 \times 10^{30\pm 2}$ yr, $\alpha_0 = 0.0266$, $M_{W\pm} = 81.24$ GeV, $M_Z = 91.45$ GeV GeV, and $M_X = 4.28 \times 10^{14}$ GeV. If the number of chiral generations is equal to four, we have

$$b_3 = 15/3;\ b_2 = 1;\ b_1 = -93/15;\ \sin^2\theta_W = 0{,}2073;$$
$$\tau_p = 1{,}7 \cdot 10^{31\pm 2}\ \text{yr};\ a_0 = 0.0324;\ M_X = 8.6 \cdot 10^{14}\ \text{GeV};$$
$$M_{W\pm} = 81{,}92\ \text{GeV};\ M_Z = 92.01\ \text{GeV}.$$

3. Supersymmetric SU(5) Model

The supersymmetric SU(5) model proposed[1] by one of the authors (E.S.F.) (Ref. 7) is a natural generalization of the asymptotically free SU(5) model. This model has the following multiplet composition: (a) one vector supermultiplet (24), which contains 24-multiplet vector particles and 24-multiplet Majorana spinor particles; (b) f generations of chiral supermultiplets of matter, each of which is comprised of $\bar{5}$ and 10 supermultiplets (each supermultiplet in turn is comprised of a complex Bose field and chiral Fermi field); spontaneous internal symmetry breaking is accomplished by Higgs supermultiplets, by one 24-supermultiplet and by several 5 and $\bar{5}$ supermultiplets. Spontaneous breaking of the original SU(5) symmetry to the symmetry $S^cU(3)\times SU(2)\times U(1)$ without supersymmetry breaking is achieved because of the presence of a nonvanishing average Higgs supermultiplet (24); the entire Higgs 24-multiplet and leptoquarks of the vector 24-multiplet, as well as their supersymmetric partners acquire a large mass M_X. To prevent an anomalously rapid decay of the proton, the color components (3,1) and $(\bar{3},1)$ of the Higgs 5 and $\bar{5}$ supermultiplets are made extra heavy (because of the interaction with the 24-multiplet of the Higgs particles).

Thus we have, up to spontaneous supersymmetry breaking (at energies of the order of 100–1000 GeV), a supersymmetric standard model $S^cU(3)\times SU(2)\times U(1)$ with f generations of supermatter and several complex superdoublets (1,2) and $(1,\bar{2})$. Since we are dealing with supermultiplets, the corresponding b_n coefficients acquire the following values: $b_3 = 9-2f$, $b_2 = 6-2f-(N_f/2)$, $b_1 = -2f-(3/10)N_f$, where N_f is the number of light superdoublets.

Specifically, when $f=3$ and $N_f=2$, we have

$$\sin^2\theta_W = 0.233;\quad M_{W^\pm} = 77.36\ \text{GeV};\quad M_Z = 88.3\ \text{GeV}$$

$$M_X = 1.2\cdot 10^{16}\ \text{GeV};\quad \tau_p = 285\times 10^{30\pm 2}\ \text{yr};\quad \alpha_0 = 0.041.$$

As we can see, although supersymmetry leads to an increase of M_X and the proton lifetime, this effect can be neutralized by increasing the number N_f of Higgs superdoublets. In fact, when $f=3$ and $N_f=4$, we have

$$\sin^2\theta_W = 0.255;\quad M_{W^\pm} = 73.94\ \text{GeV};\quad M_Z = 85.63\ \text{GeV};$$

$$M_X = 6.3\times 10^{14}\ \text{GeV};\quad \tau_p = 2.2\times 10^{30\pm 2}\ \text{yr};\quad \alpha_0 = 0.044.$$

We should note that asymptotic freedom implies that the number of supergenerations cannot be greater than four and makes it impossible to introduce the Higgs 45-supermultiplet.

[1)] A similar model has been recently proposed in a preprint by Dimopoulos and Georgi.

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