The effect of the discrete nature of a finite disordered system on the distribution of resistances

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The distribution of the resistances $w(\rho, x)$ of a disordered section of length x changes sharply near the commensurability point of the wavelength λ of an electron and the lattice constant d, when $\lambda/4 = d$. An exact expression for w is derived and the power-law asymptotic behavior, $\langle T\rangle \propto x^{-1/2}$, for the average transmission coefficient, is determined.

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The electronic states in a one-dimensional, disordered system (DS) are localized, and the static conductivity of a DS is equal to zero.[1,2] In a number of papers[3–5] the question of the resistance R of a DS of finite length L was raised. The quantity $R(L)$ is related to the Fermi momentum p_F and to the cross section S of a DS by the relation

$$R(L) = \frac{6\pi^2}{e^2 p_F^2 S}\,\rho(L), \quad \rho(L) = 1/T(L) - 1, \tag{1}$$

where T is the transmission coefficient of a DS.[3,4]

The quantity $\rho(L)$ has different values for different random potentials of a DS, and the problem of the distribution of the resistances $w(\rho, L)$ corresponding to the given characteristics of the potential must be resolved. It was shown that w decreases slowly with increasing ρ, so that $\ln\rho$ rather than ρ is self-averaging; here $\langle\ln\rho\rangle \propto L$[4,6]. This problem was investigated numerically in a number of papers.[7,8] An explicit expression for $w(\rho, x)$ in the limit $x \equiv L/l \gg 1$ (l is the mean free path) was derived by us[9] for weak scattering by random impurities (preliminary results were published in Ref. 10).

In general, it does not matter whether the impurities are distributed randomly or are situated in the discrete-lattice sites. The situation changes drastically only when the electron wavelength $\lambda = 2\pi/p_F$ is commensurate with the lattice constant d, i.e., when $\lambda/4 = d$. It has been established that if $|4p_F d - 2\pi| \ll d/l$ with $\exp(4ip_F d) \approx 1$ the density of electronic states has a singularity,[11,12] and the frequency and temperature dependences of the conductivity of an infinite DS change dramatically.[12] It was shown in Ref. 13 that the localization length increases as $\ln^2 s$ with decreasing distance s to the middle of the zone [see Eq. (5)]. We note that Dyson[14] was the first to point out the existence of commensurability effects in disordered systems.

In this letter we give an exact solution of $w(\rho, x)$ for the case $\lambda/4 = d$ and show that the transmission coefficient of a DS decreases as $x^{-1/2}$ along its length, in contrast to an exponential decay, as is generally the case.

Let us assume that a DS occupies a section $(0, L)$, $L \equiv Nd$. As in the previous case,[9] we shall use the transition-matrix formalism[6] and after simple computations reduce the calculation of the resistance ρ_N to the recurrence relations.

$$A_N = A_{N-1}(1 + 2\beta_N^2) + \beta_N(B_{N-1}e^{-i\gamma} + B_{N-1}^* e^{i\gamma}); \quad \gamma = 2p_F d$$
$$B_N = B_{N-1}e^{-i\gamma} + 2\beta_N A_{N-1} e^{-i\gamma} + \beta_N^2 (B_{N-1}e^{-i\gamma} + B_{N-1}e^{i\gamma}). \tag{2}$$

$$A_N^2 - |B_N|^2 = 1; \quad A_0 = 1; \quad B_0 = 0; \quad \rho_N = (A_N - 1)/2. \tag{3}$$

The distribution of β_k for all impurities is identical and mutually independent,

$$< \beta_k > = 0; \quad < \beta_k \beta_{k'} > = d/l\, \delta_{kk'}; \quad d/l << 1. \tag{4}$$

The averaging of the first relation in (2) shows that the average resistance is independent of y and is always identical with the result obtained for the continuous model.[3] The calculation of $\langle\rho^2\rangle$ requires that the recurrence equations be solved for the values $\langle A^2\rangle$, $\langle B^2\rangle$, and $\langle B^{*2}\rangle$. The asymptotic behavior of $\langle\rho^2\rangle$ such as $\exp[\Lambda(s)x]$ is determined by the largest root of the cubic equation

$$\Lambda^3 - 10\Lambda^2 + \Lambda(16 + s^2) - 6s^2 = 0; \quad s = (4p_F - 2\pi/d)l. \tag{5}$$

In the special cases we have

$$\Lambda = 8 - s^2/24, \quad s^2 << 1; \quad \Lambda = 7; \; s^2 \doteq 45; \quad \Lambda = 6 + 48/s^2; \quad s^2 >> 1. \tag{6}$$

In the limit $s = 0$, after incorporating Eq. (3), Eqs. (2) become one equation,

$$A_N = A_{N-1}(1 + 2\beta_N^2) + 2\beta_N(1 - A_{N-1}^2)^{1/2}, \tag{7}$$

from which it follows that

$$d < A^n > /dx = 2n^2 < A^n > - 2n(n - 1) < A^{n-2} >. \tag{8}$$

This set of equations is equivalent to the Fokker-Planck equation

$$\frac{\partial}{\partial x} w(A, x) = 2\frac{\partial}{\partial A}\left[A + (A^2 - 1)\frac{\partial}{\partial A}\right] w(A, x), \tag{9}$$

whose solution is found in explicit form

$$w(A, x) = (2\pi x)^{-1/2}(A^2 - 1)^{-1/2}\exp[-(\operatorname{arch} A)^2/8x]; \quad A = 2\rho + 1. \tag{10}$$

Using Eq. (10), we calculate the asymptotic behavior of the degree of resistance and conductivity for $s = 0$ and $x \gg 1$,

$$< \rho^\nu > = 4^{-\nu}\exp(2\nu^2 x); \quad \nu > 0, \tag{11}$$

$$< \sigma^\nu > \equiv < \rho^{-\nu} > = (2\pi x)^{-1/2} B(-\nu + 1/2, \nu); \quad 1/2 > \nu > 0,$$

where B is the Euler function. We recall that as $|s|\to\infty$, i.e., in the continuous model,[9,10]

$$\langle \sigma^{\nu} \rangle \propto \exp(-\nu(1-\nu)x); \qquad 1/2 > \nu > 0, \tag{12}$$

$$\langle \sigma^{\mu} \rangle \propto \exp(-x/4); \qquad 1 > \nu > 1/2.$$

A comparison of Eqs. (12) and (11) shows that as a result of transition from $s^2=\infty$ to $s=0$ the asymptotic behavior of $\langle \sigma^{\nu} \rangle$ changes from an exponential to a power form in the range $\frac{1}{2}>\nu>0$ and the average $\langle \sigma^{\nu} \rangle$ diverge at $\nu>\frac{1}{2}$ in the case of commensurability.

A power-law asymptotic behavior of the resistance at $s=0$ such as $\langle \rho \rangle \propto x^2$ was recently corroborated by Azbel.[15] The calculation showed that only the average $\langle \sigma^{\nu} \rangle$ of the power-law conductivity changes qualitatively as $s^2 \to 0$.

The power-law decrease of the average transmission coefficient

$$\langle T \rangle \equiv \langle 2(A+1)^{-1} \rangle \approx 2^{1/2}\pi^{-1/2}x^{-1/2}; \; x \gg 1, \tag{13}$$

is in qualitative agreement with the asymptotic behavior of $x^{-3/2}$ for the density correlator, which was determined previously.[12]

Thus the discrete nature of the system and the chaotic nature of the potential affect in a unique way the average values such as T. In an ordered system, when $\beta=\text{const}$ and $\lambda/4=d$ corresponds to the energy at the center of the forbidden band of width $\sim|\beta|$, we have $T\approx 1$ for $\beta s \gg 1$ and $T \propto \exp(-2\beta x/d)$ for $\beta s \ll 1$ when $\lambda/4=d$. If, however, β is a fluctuating potential with an average value in (4) equal to zero, then $T\propto \exp(-x/4)$ for $s^2 \gg 1$ and $T \propto x^{-1/2}$ for $s=0$. We see that a transition from a regular potential to a random potential, in general, decreases T exponentially and increases it exponentially for the states in the center of the band.

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