Gauge hierarchies in the model of constituent quarks and leptons

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The hierarchy of mass scales in the grand unified models finds a natural embodiment in the preon model. The first scale corresponds to the size of the constituent particles; the second arises from the first as a result of a gravitational transformation of preons with nonconserved preon number.

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All of the known grand unified models[1)] include two basic sets of scalar Higgs multiplets. The fields ϕ^{α}_{β} of the regular representation of the "grand" group $G(n)$ generate large vacuum expectations $V \sim \mathcal{O}(10^{15})$ GeV and lead to a decomposition of this group without a lowering of the rank n:

$$G(n) \to G_1(n) \equiv SU(3)_c \oplus SU(2) \oplus U(1) \oplus G(n-4)_g, \tag{1}$$

where $G(n-4)_g$ is some family group or horizontal group. The fields H ($H_{\alpha\beta}$, $H^{\delta}_{\alpha\beta\gamma}, \ldots$) transform under various complex representations of G and, by assumption, generate small expectations [from $v_f \sim \mathcal{O}(10^2)$ GeV for the weak-interaction sector to $v_g \gtrsim \mathcal{O}(10^5)$ GeV for the mass scale of flavor-changing horizontal bosons] and lead to a further breaking of G, but in this case with a lowering of the rank n:

$$G_1(n) \to G_2(3) \equiv SU(3)_c \oplus U(1)_{EM}. \tag{2}$$

In the standard approach there is no difference between the fields ϕ and H. They interact with each other (in a "bare" fashion and/or through the exchange of gauge fields) and would have to generate vacuum expectations $v \sim V$ of identical order of magnitude; this could be avoided only through an unnatural fitting of the parameters in the corresponding Higgs polynomial.[1] A profound difference arises between these fields, however, in the constituent composite model of quarks, leptons, and Higgs scalars.[2,3]

For definiteness, we will consider the preon model,[3] which is closest in spirit to SU(5): five spinors of the fundamental SU(5) representation, $\mathscr{P}_i, i = 1, \ldots, 5$ (the chromons $\mathscr{P}_c, c = 1, 2, 3$, and the flavons $\mathscr{P}_f, f = 1, 2$), plus a certain number of "family" preons $\mathscr{P}_g, g = 1, \ldots$ [generons with electric charges $Q_g = 0$; in SU(8), $g = 1, 2, 3$]. Metacolor forces act between the preons; these forces are analogous to ordinary color, but at distances R_{MC} in the range 10^{-14} GeV$^{-1} \gtrsim R_{MC} \gtrsim R_p = 10^{-19}$ GeV^{-1} and with the chiral symmetry group $G_{MC} = SO(3)_L \oplus SO(3)_R$, so that the preons remain massless. In this model the scalar fields ϕ and H are composite fields and differ in preon number N($\widetilde{\mathscr{P}}$ is an antipreon):

$$\phi^{\alpha}_{\beta} \sim (\tilde{\mathscr{P}}^{\alpha} \mathscr{P}_{\beta}), \quad N = 0; \quad H_{\alpha\beta} \sim (\mathscr{P}_{\alpha} \mathscr{P}_{\beta}), \quad H^{\delta \cdots}_{\alpha\beta\gamma\ldots} \sim (\tilde{\mathscr{P}}^{\delta} \mathscr{P}_{\alpha} \mathscr{P}_{\beta} \mathscr{P}_{\gamma}) \,. \tag{3}$$

where $\alpha, \beta, \gamma = (i, g) = 1, \ldots, 5, \ldots, n+1$. In addition to these fields, the quarks, the leptons, and (possibly) the gauge fields are also composite (more on this below).

The fields ϕ "organize" the basic condensate of the system [in the $SU(5) \oplus G(n-4)_g$ components]:

$$< \phi >_{\circ} \sim < \tilde{\mathscr{P}} \mathscr{P} >_{\circ} = (1.1)_{\circ} + (24.1)_{\circ} \sim V. \tag{4}$$

We assume that the fields H, in contrast with the fields ϕ, do not condense, because of a nonzero preon number; the situation is analogous to that in quantum chromodynamics, where the dibaryons do not condense, $\langle BB \rangle_0 = 0$. As for baryons, however, there may be a gravitational annihilation[4,5] for the preons (and thus for the fields H) in processes involving virtual "mini" black holes (with a Planckian mass, $M \sim 1/R_p$):

$$\mathscr{P}\mathscr{P} + D_{M,J,Z} \to D_{M',J',Z'} \to D_{M,J,Z} + \tilde{\mathscr{P}}\mathscr{P}, \tag{5}$$

where M is the mass of the hole, J is the total angular momentum, and Z is the set of all the gauge charges (either exact charges or charges which break spontaneously at distances $R > R_p$). The processes in (5) effectively lead to the field transformation $H_{\alpha\beta} \to \mathrm{Sp}\phi^2$ and hence to the condensation of those components (α, β) of the H multiplet which do not have gauge charges at Planckian distances. We thus arrive at the concept of composite (over distances $R > R_p$) gauge fields for the weak and horizontal interactions. If all the gauge fields are assumed to be composite, then the $SU(3)_c \oplus U(1)_{EM}$-nonsinglet components of the field multiplets also acquire vacuum expectations as a result of transformation (5), and we have an analogous nonconservation of color and electric charge. If, instead, all the gauge fields are elementary, the transformation in (5) will not occur at all [since all the gauge charges of the $(\mathscr{P}\mathscr{P})$ and $(\tilde{\mathscr{P}}\mathscr{P})$ systems will always by distinct], and the fields H will have no vacuum expectations.

To evaluate the quantity $\nu \equiv \langle H \rangle_0 \sim \langle \mathscr{P}\mathscr{P} \rangle_0$, we assume that the probability W for the annihilation (5) is determined by the "dimensions" of the black hole,[4] and the correct power of the ratio R_p/R_{MC} in W follows from the metasinglet, effective, four-preon Lagrangian, which leads to $\Delta N = 2$ for the chiral preons $\mathscr{P}_{L,K}$ (with the coupling constant of minimum dimensionality):

$$\mathscr{L} \sim \frac{1}{M^3} (\mathscr{P} C \mathscr{P})(\bar{\mathscr{P}} i \gamma_{\mu} \partial_{\mu} \mathscr{P}) + h.c., \qquad C = i\gamma_2 \gamma_{\circ} \,. \tag{6}$$

This coupling leads to terms with $\Delta N = 2$, which are linear in the $SU(3)_c \oplus U(1)_{EM}$-singlet components $(\alpha, b, c, d, \ldots)$ of the fields H in the Higgs polynomial in the "composite" quark–lepton Lagrangian:

$$P(H, \phi) = P_{\circ} + \eta H_{ab} + \xi H^{d}_{abc} + \ldots \,. \tag{7}$$

Here, in accordance with Lagrangian (6), we must set $\eta \sim \xi \sim (R_p/R_{MC})^3 \; 1/R^3_{MC}$, assuming that the dimensions of the coupled states are $\sim R_{MC}$. The quantity P_0 is the conventional symmetric polynomial of the fields ϕ and H (Ref. 1), which generates large ($V \sim 1/R_{MC}$) vacuum expectations for the fields ϕ but which does not condense [if the signs of the constants of the (ϕH) cross terms are chosen appropriately]. For the fields H we then find from (7)

$$v \approx (R_P/R_{MC})^3 V \approx 10^{-12} V, \tag{8}$$

if $R_{MC} \approx 10^{-15}$ GeV^{-1}, which is very good for the mass scale of the weak interactions v_f but unacceptably small for the mass scale of the "nondiagonal" horizontal bosons v_g. The gravitational mechanism in (5) does not distinguish the "nongauge" flavor ($f = 2$) and horizontal ($g = 1, 2, 3, \ldots$) components of the fields H, inducing an identical condensation of these components. It appears to us, however, that there is still the possibility that the v_g scale is higher than v_f: In the polynomial in (7) the terms linear in the fields have the SU(5) symmetry if the indices, $a, b, \ldots$ take on only the "horizontal" g values, or they have the SU(4) symmetry if at least one of the indices takes on the value $f = 2$. Since these terms clearly violate the renormalizability (they are G-noninvariant), they acquire large corrections, proportional to powers of $1/R_{MC}$, at distances $R > R_{MC}$, so that the effective values of the constants $\eta, \xi, \ldots$ are reduced substantially. The corrections to the terms containing the fields H with the (fg) components will be large (because of the lower symmetry) and will—it may be hoped—ultimately lead to the necessary distinction between v_f and v_g with an appropriate choice of R_{MC}. As may be seen from the same polynomial [in (7)], the fields H, despite the small vacuum expectations, acquire large masses ($\mu_H \sim V$), because of an interaction with the fields Φ, so that not even a single light scalar is left in the electroweak sector.

We have found a natural embodiment of the hierarchy of vacuum expectations $v \lll V$ in grand unified models. This embodiment corresponds to an approximate conservation of preon number N in the composite model of quarks, leptons, Higgs scalars, and massive gauge fields, and it necessarily leads to strong interactions in the weak processes of quarks and leptons at energies above the enregy of the unitary limit, $E > G_F^{-1/2}$. At low energies, this effect is seen only in the logarithmic dependence on μ_H^2 (Ref. 6) and hence is small. In our case, for the mass $\mu_H \sim 10^{15}$ GeV we would have to have a total cross section $\sigma_{\nu_\mu e}$ some 6% lower than the standard value. On the other hand, if, as a result of the strong interaction between the weak bosons, a Weinberg-Salam bound scalar with a mass $\sim \mathcal{O}(1)$ TeV arises, then there will be essentially no decrease in $\sigma_{\nu_\mu e}$, and we will remain completely ignorant up to the energy of the unitary limit.

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[1]We will be thinking of both grand unified models with a single quark-lepton generation and (primarily) models with three generations and a local SU($n \geqslant 8$) symmetry. The necessary details and references are given in the review articles in Refs. 1 and 2.

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