Gyromagnetic rations and nature of back bending of the moment of inertia

Yu. T. Grin'
I. V. Kurchatov Institute of Atomic Energy
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It is shown that the gyromagnetic ratio for the rotational excitations take on values that differ significantly from Z/A in the case when the nature of the back bending of the moment of inertia is connected with intersection with the state of broken pair of nucleons at a level with angular momentum $j = \ell + 1/2$. The dependence of g on the nuclear spin is calculated.

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Two possible causes of back bending of the moment of inertia of the atomic nucleus[1] are presently under intensive study: 1) a jumplike transition from the superconducting to the normal state, 2) a transition to a state with one broken pair, the angular momentum of which is directed along the rotation axis. In this article we estimate the behavior of the gyromagnetic ratio at different angular momenta I for the indicated causes of the back bending; this estimate yields additional information on the nature of the back bending and permits a choice between the two possibilities.

The abrupt change of the pair correlation Δ leads to a jump in the moment of inertia J, but alters little the collective gyromagnetic ratio $g_R = J_Z/(J_Z + J_N)$. The expression for the moments of inertia of nucleon systems (the subscripts Z and N pertain, respectively, to protons and neutrons) can be represented in the form[2] $J_i = J_0(N_i/A)\phi(\kappa_i)$, where J_0 is the rigid-body moment of inertia, N_i is the number of nucleons of sort i (N or Z), $\phi(\kappa)$ is a universal function of the parameter $\kappa_i = \omega_0\beta/2\Delta_i$ and is tabulated in[2], β is the deformation of the nucleus, and the oscillator frequency is $\omega_0 = 41/A^{1/3}$ MeV. Substitution of the characteristic values of β and Δ_i for the region of the rare-earth nuclei yields $g_R \approx 0.3$ to 0.35, which is in good agreement with experiment at small nuclear angular momenta I. Owing to the difference between the values of Δ for neutrons and protons, the jumplike vanishing of Δ under the influence of rotation occurs first for the neutrons and only then for the protons. It must be recognized here that at the point $I_{cr} = 12$ to 16 (the point where Δ_N vanishes) the initial value of Δ_Z was already decreased at least 40% by the Coriolis forces.[3] It is easy to verify that an increase of I leads in this case first to a slight decrease of the gyromagnetic ratio, to a value $g \approx 0.28$, but when Δ_Z vanishes completely we have an increase to $g_R = Z/A = 0.4$. Thus, the gyromagnetic ratio in this case changes very little with increasing angular momentum also at the points of phase transition.

The situation is entirely different when the back bending is due to pair breaking and with alignment of the angular momenta of the particles along I under the influence of rotation. In this case the angular momentum I and the magnetic moment each consist of two parts—collective and quasiparticle—corresponding to the angular momenta of the particles of the broken pair. When the collective and quasiparticle magnetic moments are of opposite sign, a strong decrease of the total magnetic moment is possible, and when the signs are equal a strong increase of g can take place. Pair breaking is energywise favored in a state with a large angular momentum $j \gg I$. It appears that in the region $A \sim 150$–190, $N \sim 90$–106, the neutron pair breaks at the level $i_{13/2}$, and in the region $A \sim 126$ pair breaking takes place at the level $h_{11/2}$. Calculation of the back bending in general form is very difficult because it is impossible to carry out a joint analysis of the rotation, pair correlation, and deformed self-consistent field. We shall therefore consider a simple model[4] that contains all the characteristic features of the back bending: a nuclear core rotating with angular momentum $\bar{R}$ and having a moment of inertia J, and a degenerate j-level with two (or several) nucleons that simulate a pair of particles with angular momentum $\bar{K}$ (the maximum possible angular momentum is $K_{max} = 2j - 1$). In this case, the gyromagnetic ratio is

$$g = g_R + (g_p - g_R)\frac{K}{I}, \tag{1}$$

where the quasiparticle gyromagnetic ratio is $g_p = g_l \pm (g_s - g_l)/(2l+1)$ for $j = l \pm 1/2$, and g_R is the usual collective gyromagnetic ratio. For protons we have $g_l = 1$ and $g_s = 5.586$, while for neutrons $g_l = 0$ and $g_s = -3.83$. Back bending occurs when the levels of the ground rotational band with $K = 0$ and energy $E_0(I)$ cross the levels of the two-quasiparticle band with $K = K_{max} = 2j - 1$ and $E_K(I)$ at a certain I_{0K}.

At the crossing point, the gyromagnetic ratio changes jumpwise from the value $g_R(K=0)$ to the value of g given by formula (1) with $K = K_{max} = 2j - 1$. It follows from (1) that a strong change of g is possible near the back-bending point for a pair at a level with $j = l + 1/2$, whereas for a pair at the level $j = l - 1/2$ the change of g is small ($j \gg 1$). In the case of breaking of a neutron pair, it decreases strongly and may even become negative, and in the case of a proton pair, to the contrary, it increases strongly.

Actually, there is always an interaction between the levels, and a "pseudocrossing" takes place. As a result, the wave function of the yrast-states is a superposition of a state without a broken pair and with a broken pair. In this case the expression for g takes the form

$$g = g_R + b_I^2(g_p - g_R)\frac{K}{I}, \tag{2}$$

where b_I^2 is the square of the amplitude of the impurity state, which is determined from the formulas for the wave functions of the two crossing levels,

$l_j\ N_j$	$x \backslash I$	2	4	6	8	10	12	14	16	18	20
$i_{13/2}$, N	0	0,30	0,30	0,30	0.30	0.30	0,30	-0,20	-0,15	-0,10	-0,06
	0.2	0.30	0.30	0,29	0,26	0,20	0,0	-0,10	-0,10	-0,04	-0,04
	0,5	0,30	0.27	0,23	0,17	0,08	0,0	-0,02	-0,02	-0,01	+0,01
	1.0	0,26	0,25	0,13	0.06	0.01	0,0	0,01	0,02	0,04	0,06
$h_{11/2}$, Z	0,2	0,44	0,46	0,50	0,61	1,19	1,46	1,38	1,32	1,2	1,16
	0,5	0,47	0,53	0,69	0,72	1,2	1.25	1.25	1,2	1,1	1,08

$$b_l^2 = \frac{[\sqrt{1+\alpha^2} + \mathrm{sgn}(E_0 - E_k)]^2}{\alpha^2 + [\sqrt{1+\alpha^2} + \mathrm{sgn}(E_0 - E_k)]^2} \quad (3)$$

Here $\alpha = 2JV_{12}/(2j-1)(I_{0K} - I)$, and the interaction takes the Coriolis form $V_{12} = -(2j-1)\ Ix/2J$ with an arbitrary parameter x that takes into account the presence of deformation in the nucleus. The table lists the values of the gyromagnetic ratios calculated from formula (2) as functions of I for different values of the parameter x for a neutron pair at the $i_{13/2}$ level and a proton pair at the $h_{11/2}$ level. The quantity $x=0$ corresponds to noninteracting levels. For real nuclei having back bending, the values of x lie in the interval $0.2 \leqslant x \leqslant 0.5$, and as $x \to 1$ the levels are strongly intermixed and the back bending vanishes.

Thus, if the cause of the back bending is quasi-intersection with the level of a broken pair with large angular momentum, then a strong change takes place in the gyromagnetic ratio (if $j = l + 1/2$).

At the present time there are only measurements of the averaged value of g in the interval $8 \leqslant I \leqslant 16$, namely $\bar{g}(^{168}\mathrm{Hf}) = 0.07 \pm 0.04$ and $\bar{g}(^{172}\mathrm{Hf}) = 0.14 \pm 0.0.$ [5] So strong a decrease of g seems to offer evidence in favor of neutron-pair breaking at the $i_{13/2}$ level as being the cause of the back bending in the rare-earth region. In addition, the change of the value of the gyromagnetic ratio helps explain the isotopic nature of the broken pair and the angular momentum of its level (owing to the difference in the behavior on the levels with $j = l \pm 1/2$). A more accurate experimental study for each I (both for nuclei with and without back bending) is very desirable, since it helps explain the question of the level mixing.

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