Hydrodynamic limit for volume oscillations of the nucleus in the theory of finite Fermi systems

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(Submitted June 2, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **22**, No. 2, 114–117 (July 20, 1975)

A hydrodynamic limit corresponding to volume oscillations of the nucleus is obtained in the theory of finite Fermi systems. A nuclear analog of the Cerenkov, namely generation of a density shock wave in nuclear reactions, is discussed.

PACS numbers: 21.60.

In this article we wish to call attention to a possible hydrodynamic limit in the microscopic theory of the oscillations of a nucleus. The main physical result of the paper is the following: collective oscillations of the nucleus, with high frequencies ω, constitute volume waves and are described by an equation of the hydrodynamic type for an effective field $V(\mathbf{x})$[1]

$$\left(\Delta + \frac{\omega^2}{c^2}\right)V(\mathbf{x}) = 0 \quad \text{(inside the nucleus)} \tag{1}$$

with a boundary condition on the surface Σ

$$\left.\frac{\partial V}{\partial \mathbf{x}}\right|_{\Sigma} = 0\,. \tag{2}$$

To demonstrate this, we write down an equation for the matrix elements $V_{11'}$ over the one-particle states[1]

$$V_{11'}(\omega) = V^{\circ}_{11'}(\omega) + \sum_{22'} \langle 12' | b | 21' \rangle \frac{n_2 - n_{2'}}{\epsilon_2 - \epsilon_{2'} + \omega} V_{22'}(\omega). \quad (3)$$

Here $\langle 12' | b | 2' \rangle$, n_2, and ϵ_2 are the interaction, the occupation numbers, and the energy of the quasiparticles, respectively, while $V^0_{11'}(\omega)$ is the unrenormalized field. We consider the natural solutions ($V^0 = 0$) of (3) with frequencies in the interval ($\epsilon_F = p_F^2/2$ is the Fermi energy and p_F is the Fermi momentum)

$$\epsilon_F >> \omega >> | \epsilon_2 - \epsilon_2 | . \quad (4)$$

Integrating (3) with respect to $1/\omega$, we obtain

$$V(1) = -\frac{1}{\omega^2} Tr_2\Big(G_{12}[\rho(2); [S(2); V(2)]]\Big) + \frac{1}{\omega} Tr_2(G_{12}[\rho_2; V_2]),$$

where ρ is the one-particle density matrix and S is the self-consistent Hamiltonian. Assuming for simplicity the interaction G to be a function of the spatial coordinates only, we have ($\hbar = m = 1$, $\rho(\mathbf{x}) \equiv \rho(\mathbf{x} | \mathbf{x}')$ is the density in the nucleus)

$$-\omega^2 V(\mathbf{x}) = \int d\mathbf{x}' G(\mathbf{x} | \mathbf{x}') \operatorname{div}(\rho(\mathbf{x}') \nabla V(\mathbf{x}')). \quad (5)$$

Bearing in mind the fact that undamped volume waves exist only in the case of repulsion, we use the simplest approximation of the interaction in the theory of Finite Fermi systems ($n = p_F^3/3\pi^2$, f is the coupling constant)

$$G(\mathbf{x} | \mathbf{x}') = f \left| \frac{dn}{d\epsilon_F} \right. \delta(\mathbf{x} - \mathbf{x}'). \quad (6)$$

Substituting (6) in (5) and integrating the obtained equation over a layer of thickness L($R_0 \gg L \gg a$, R_0 is the radius of the nucleus, and a is the diffuseness parameter) near the edge of the nucleus, we obtain directly (1) and (2). For the speed of sound c we have

$$c = p_F \sqrt{f/3}.$$

The result has a lucid physical meaning. The inequalities (4) mean that the correlation length p_F/ω should be in the interval (r_0 is the distance between particles)

$$R_0 >> p_F/\omega >> r_0 \sim 1/p_F , \quad (7)$$

which in essence is equivalent to the requirement that the collective motion be local. We note that the condition (7), physically speaking, is analogous to the criterion for the transition, in hydrodynamics, to a finite Fermi system with pairing.[2]

The equations derived in the paper are, in our opinion, of more than just methodological interest. Let us consider the problem posed in[3], that of the excitation of volume oscillations in scattering of high-energy nucleons. The unrenormalized field V^0 is in this case of the form

$$V^{\circ}(1) = Tr_2(G_{12} \delta\rho(2)),$$

where $\delta\rho(2)$ is the distortion of the density matrix inside the nucleus and is due to the incident particle. Neglecting the bending of its trajectory and changing over to the time-dependent description, we obtain

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) V(\mathbf{x}; t) = -f \Big/ \frac{dn}{d\epsilon_F} \frac{\partial^2}{\partial t^2} \delta(\mathbf{x} - \mathbf{b} - \mathbf{v}t) \frac{\rho(\mathbf{x})}{c^2 \rho(0)} , \quad (8)$$

where b is the impact parameter, $\mathbf{v}$ is the velocity of the outer nucleon, and the last factor in the right-hand side of (8) reflects the fact that the perturbation is different from zero inside the nucleus. Equation (8) coincides with the wave equation for the potentials in classical hydrodynamics, with a source

$$-f \Big/ \frac{dn}{d\epsilon_F} \frac{\partial^2}{\partial t^2} \delta(\mathbf{x} - \mathbf{b} - \mathbf{v}t) \frac{\rho(\mathbf{x})}{c^2 \rho(0)}$$

This analogy allows us to draw a number of physical conclusions without solving the equation. In particular, if the velocity of the outer particle is $v > c$, then we get the analog of the Cerenkov wave, i.e., a density "shock wave" of sorts. We note that the "transition radiation" due to capture of an external nucleon was considered in[4].

A FEW REMARKS AND CONCLUSION

For the assumptions (4) to be compatible with the values of ω determined from (1) and (2) it is necessary to satisfy the inequality $A^{1/3} \gg c/p_F \gg 1$ (A is the atomic number). This signifies a rather narrow region of applicability of the hydrodynamic description of the volume oscillations.

A direct analogy with the Cerenkov effect is meaningful only in the case when the dimensions $c\tau$ of the radiation cone (τ is the time of flight of the external particle through the nucleus) lies in the interval $R_0 \gtrsim c\tau \gg r_0$.

The interaction G near the edge of the nucleus reverses sign,[1] as a result of which the volume mode attenuates on the surface and should match there the surface oscillations.[5]

The volume oscillations were calculated in a number of papers (see, e.g.,[6]) with the aid of the equations of the homogeneous Fermi liquid with the boundary condition

$$V\Big|_\Sigma + \text{const} \frac{\partial V}{\partial n}\Big|_\Sigma = 0,$$

i.e., with introduction of a free parameter. We emphasize that the microscopic theory provides a unique condition on the surface (2), and in the region where hydrodynamics is valid it leads to much simpler equations than in[6].

The author is indebted to S.T. Belyaev and V.G. Zelevinskiĭ for valuable critical remarks, and also to Ya.S. Derbenev and V.B. Telitsyn for useful discussions.

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