Feasibility of observing effects of order G^2 in $K^+ \to \pi^+ \ell^+ \ell^-$ decays

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It is shown that the ratio of the spectra of the $K^+ \to \pi^+\mu^+\mu^-$ and $K^+ \to \pi^+ e^+ e^-$ decays is quite sensitive to the contribution of the weak interaction of second order in G.

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The question of the magnitude of effects of second order in the weak interaction is decisive for the determination of the structure of the weak interaction. The most important results in this direction were obtained by analyzing the effect of the K_L and K_S mass difference and of the $K_L \to \bar{\mu}\mu$ decay.[1,2] However, the possible influence of strong interactions in the first case (noted in[1]) and the possibility of cancellation of the contributions of order G^2 and $G\alpha^2$ in the real part of the $K^0 \to \bar{\mu}\mu$ decay amplitudes introduces an uncertainty in the conclusions obtained by analyzing these effects. The purpose of this article is to call attention of the experimenters to a simple possibility of revealing the role of effects of second order in G by investigating the spectra of the $K^+ \to \pi e^+ e^-$ and $K^+ \to \pi^+\mu^+\mu^-$ decays.

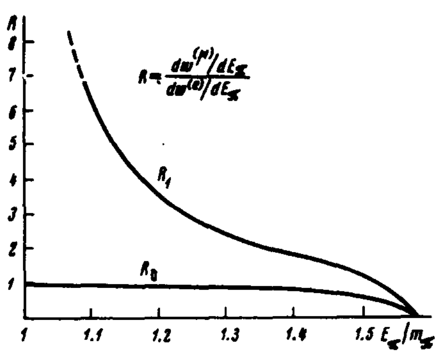
It is usually assumed that these decays are due to the action of weak and electromagnetic interaction, and should have, according to the estimates,[3,4] a probability on the order of $10^{-6} - 10^{-7}$ of the total probability of the K^+-meson decay. The invariant amplitude of the $K^+ \to \pi l^+ l^-$ decay takes in this case the form

$$\mathcal{M}^{(\gamma)} = a(p_K + p_\pi)_\alpha \bar{u}_l - \gamma_\alpha v_{l^+} . \qquad (1)$$

In the presence of weak interaction of second order, an additional part appears

$$\mathcal{M}^{(W)} = [b(p_K + p_\pi)_\alpha + c(p_K - p_\pi)_\alpha]\bar{u}_l - \gamma_\alpha(1+\gamma_5)v_{l^+} . \qquad (2)$$

The quantities a, b, c in (1) and (2) are functions of the pion energy. In view of the smallness of the amplitude (1), the contribution of a weak interaction of second order in G, as noted in[5], could be noticeable. It is indicated in the same reference that the G^2 contribution can be separated by polarization measurements in the decay $K^+ \to \pi^+ l^+ l^-$. The method proposed in this article for the separation of the G^2 effects is simpler and uses a comparative analysis of the spectra of the $K \to \bar{\mu}\mu\pi$ and $K \to \bar{e}e\pi$ decays.



The point is that the ratio of the differential probabilities (with respect to the pion energy) of the $K^+ \to \pi^+\mu^+\mu^-$ and $K^+ \to \pi^+ e^+ e^-$ decays is given by the formula

$$R = \frac{dw^{(\mu)}/dE_\pi}{dw^{(e)}/dE_\pi} = \left(\frac{Q^2 - 4m_\mu^2}{Q^2}\right)^{1/2} \left\{ 1 + \frac{2m_\mu^2}{Q^2} + \frac{3m_\mu^2}{2|\mathbf{p}_\pi|^2(|a+b|^2 + |b|^2)} \times \left[bb^*\left(1 + \frac{2E_\pi}{M} + \frac{m_\pi^2}{M^2}\right) + (bc^* + cb^*)\left(1 - \frac{m_\pi^2}{M^2}\right) + cc^*\frac{Q^2}{M^2}\right]\right\} . \qquad (3)$$

where $Q^2 = (p_K - p_\pi)^2 = M^2 + m_\pi^2 - 2ME_\pi$, and M is the K-meson mass. Therefore an experimental measurement of the ratio R makes it possible to separate directly the contribution of the interaction of second order in G to the decay amplitude. The figure shows the dependence of R on E_π. The curve R_0 corresponds to $\mathcal{M}^{(W)} = 0$; the curve R_1 corresponds to the case $a = -b$ and $\xi = c/b = 0$. It is seen from the figure that in the region corresponding to the maximum of the calculated spectrum of the $K^+ \to \pi^+\mu^+\mu^-$ decay (i.e., at $E_\pi/m_\pi \approx 1.5$, see[4]) the value of R could be double the value obtained for R_1 with allowance for the second-order weak interaction. As $E_\pi/m_\pi \to 1$, the value of R_1 increases abruptly, whereas $R_0 \to 0.945$. This behavior of R_1 makes it possible, in principle, to determine very small (of order m_e^2/m_μ^2) contributions of the second-order interaction, by the decrease in the statistics as $E_\pi/m_\pi \to 1$ will impose definite limitations.

By now, 41 cases of the $K^+ \to \pi^+ e^+ e^-$ have already been observed.[6] Since the probability of the $K^+ \to \pi^+\mu^+\mu^-$ is estimated in[4] at $\approx 1/5$ of the probability of the $K^+ \to \pi^+ e^+ e^-$ decay, and the statistics have a tendency to increase rapidly, there are grounds for hoping that the proposed analysis will become performable in the near future.

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