A possible mechanism for the $\Delta T=1/2$ rule in nonleptonic decays of strange particles

A. I. Vaĭnshteĭn, V. I. Zakharov, and M. A. Shifman

Institute of Theoretical and Experimental Physics
(Submitted June 14, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **22**, No. 2, 123–126 (July 20, 1975)

It is shown that allowance for strong interactions introduces into the effective weak-interaction Hamiltonian new terms that contain both left- and right-helical particles. Estimates in the simple quark model show that the contribution of these terms to the amplitudes of the $K\to 2\pi$ and $K\to 3\pi$ decays can dominate and lead to the $\Delta T = 1/2$ rule. In this case, there are no grounds for expecting an enhancement of nonleptonic decays of charmed particles over their leptonic decays.

PACS numbers: 12.30., 13.25.D, 11.30.G

A number of interesting results were obtained recently concerning the structure of the effective weak-interaction Hamiltonian within the framework of asymptotically free theories of strong interactions. It is proposed that the weak interactions are described by gauge theories of the Weinberg-Salam type, while strong interactions are connected with exchange of an octet of massless vector gluons that interact with the color degree of freedom of the p, n, λ, and c quarks. In that case there arise, in perturbation theory in the strong-interaction constant, terms that contain $(g^2 \ln \mu_w^2/\mu^2)^n$, where μ_w is the W-boson mass and $\mu \sim m_p$ is a certain characteristic mass. The principal logarithmic corrections can be summed so as to obtain a weak-interaction Hamiltonian with account taken of the strong interactions. It was shown, in particular, that terms with $\Delta T=1/2$ in product of charged currents are enhanced by the strong interactions, while those with $\Delta T=3/2$ are weakened, although it appears that the effect is numerically not large.[1]

We shall show that allowance for strong interactions leads also to the appearance of new terms in the weak-interaction Hamiltonian

$$\Delta H_{eff} = \frac{2G_F}{\sqrt{2}} \sin\theta\cos\theta \{ A(\bar{\lambda}_L \gamma_\mu t^a n_L)(\bar{p}_R \gamma_\mu t^a p_R + \bar{n}_R \gamma_\mu t^a n_R + \bar{\lambda}_R \gamma_\mu t^a \lambda_R) + B(\bar{\lambda}_L \gamma_\mu n_L)(\bar{p}_R \gamma_\mu p_R + \bar{n}_R \gamma_\mu n_R + \bar{\lambda}_R \gamma_\mu \lambda_R) \} \tag{1}$$

where $G_F = 10^{-5} m_p^{-2}$, θ is the Cabibbo angle, p, n, and λ are the operators of the corresponding quark fields, $\psi_L \psi_R = \frac{1}{2}(1 \pm \gamma_5)\psi$, $t^a (a=1,\ldots,8)$ are 3×3 matrices act-

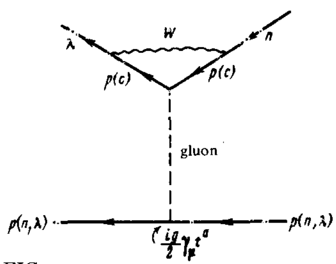

FIG.

ing in the space of the color indices, and A and B are constants.

The Hamiltonian (1) contains the $\Delta T = 1/2$ rule and, unlike in the usual case, contains both right-helical and left-helical fermions. The estimates that follow show that this circumstance can lead to predominance of the contribution (1) to the physical $K \to 2\pi$ and $K \to 3\pi$ amplitudes.

In lowest order of perturbation theory, the terms containing the interaction of right-hand particles stem from the diagram in Fig. 1. A direct calculation yields

$$A = -\frac{1}{3}\frac{g^2}{16\pi^2}\left(\ln\frac{\mu_w^2}{\mu^2} - \ln\frac{\mu_w^2}{\mu_c^2}\right), \quad B = 0, \tag{2}$$

where g is the gluon-quark coupling constant and μ_c is the mass of the charmed quark.

In the limit of exact $SU(4)$ symmetry, when the masses μ_c and μ_p are equal, expression (2) vanishes, corresponding to exact cancellation of the contributions of the c and p quarks. In the real case $\mu_c \gg \mu_p, \mu$, however, the difference between the logarithms in relation (2) is not small. In this respect we disagree with the statements made in[1], where the diagram shown in the figure has not been considered and it is stated that all the unaccounted-for diagrams make a contribution proportional to $(\mu_c^2 - \mu_p^2)/\mu_w^2$.

Using the renormalization group, we can sum in standard fashion the senior logarithmic terms and obtain the estimate

$$A \approx -6.5 \cdot 10^{-2}; \quad B \approx -1.5 \cdot 10^{-2}, \tag{3}$$

where we have assumed $\mu = 0.7$ GeV, $\mu_w = 70$ GeV, $\mu_c = 2$ GeV, and $g^2(\mu^2)/4\pi = 1$. Since there is a certain leeway in the choice of the numerical values of the masses and of the coupling constants, relation (3) can be valid accurate to a factor ~2.

We proceed now to estimate the matrix elements of the ordinary weak four-fermion interaction operator containing only left-hand particles, and of the new operator containing ψ_L and ψ_R:

$$M_1 = \langle \pi^+ | \bar{\lambda}_L \gamma_\mu n_L \bar{p}_L \gamma_\mu p_L - \bar{\lambda}_L \gamma_\mu p_L \bar{p}_L \gamma_\mu n_L | K^+ \rangle$$

$$M_2 = \langle \pi^+ | \bar{\lambda}_L \gamma_\mu t^a n_L (\bar{p}_R \gamma_\mu t^a p_R + \bar{n}_R \gamma_\mu t^a n_R + \bar{\lambda}_R \gamma_\mu t^a \lambda_R) | K^+ \rangle . \tag{4}$$

We have chosen for the analysis the matrix element of the K-π transition, since the amplitudes of all the nonleptonic K-meson decays are reducible to the amplitude of the K-π transition with the aid of the low-energy theorems and the PCAC hypothesis.

The estimate of M_1 is quite standard. If the π or K meson consists of a quark-antiquark pair, then we need take into account only the contribution of the intermediate vacuum state (in the s and t (u) channels) and reduce the matrix element M_1 to the current matrix elements or to the constants f and f_K of the $\pi - \mu\nu$ and $K \to \mu\nu$ decays. We thus obtain

$$M_1 = -\frac{2}{3}\frac{1}{4} f_\pi f_K K^2, \tag{5}$$

where account is taken also of the factors connected with the color indices.

For an analogous estimate of the matrix element M_2, we use the Fierz transformation

$$\bar{\psi}_{L1}\gamma_\mu\psi_{L2}\bar{\psi}_{R3}\gamma_\mu\psi_{R4} = -2\bar{\psi}_{L1}\psi_{R4}\bar{\psi}_{R3}\psi_{L2}$$

and the equation of motion, which reduces $\psi_L \psi_R$ to the divergence of a current, e.g.,

$$\bar{\lambda}_L p_R = \frac{-i}{\mu_\lambda + \mu_p}\partial_\mu(\bar{\lambda}_L \gamma_\mu p_L), \tag{6}$$

where μ_λ and μ_p are the masses of the λ and p quarks, respectively. We note that relation (6) is valid in the considered theory not only for the bare quarks but also for the interacting ones.

As a result we obtain

$$M_2 = -\frac{16}{9}\frac{1}{\mu_p(\mu_p + \mu_\lambda)}\frac{1}{4} f_\pi f_K m_\pi^2 m_K^2 . \tag{7}$$

The mass of the quark can be estimated by using relations of the $SU(6)$ symmetry type, which connect the matrix elements $\langle \pi^+ | \bar{p}\gamma_5 n | 0 \rangle$ and $\langle \rho^0 | J_\mu^{em} | 0 \rangle \equiv \epsilon_\mu F_\rho m_\rho$.[2] Then, for example,

$$\mu_p = \frac{m_\pi^2}{3 m_\rho}\frac{F_\pi}{F_\rho}; \quad F_\pi = \frac{f_\pi}{\sqrt{2}} \approx 0.68\, m_\pi; \quad F_\rho \approx 1.0\, m_\pi \tag{8}$$

or, numerically,[2] $\mu_p \approx 5$ MeV and $\mu_\lambda \approx 120$ MeV.

As a result we get

$$M_2 / M_1 = \frac{8}{3}\frac{m_\pi^2}{\mu_p(\mu_p + \mu_\lambda)}\frac{m_K^2}{K^2} \approx 70 \quad \text{at} \quad K^2 = m_K^2 . \tag{9}$$

The ratio (9) is large to the extent that the matrix element M_1 is suppressed: an interaction containing only left-hand particles cannot annihilate right-hand quarks in mesons (in the limit of massless quarks). This is formally expressed as smallness of the constants $f_\pi \approx m_\pi$ and $f_K \sim m_\pi$.

To estimate the contributions of various terms in the Hamiltonian it is necessary to take into account not only the ratio of the matrix elements (9), but also the coefficients with which the corresponding operators enter in the effective weak-interaction Hamiltonian. Under our assumptions concerning the values of μ, μ_c, μ_w, and $g^2(\mu^2)$, the coefficient preceding M_1 is ≈ 2.5,[2] and the coefficient of M_2 is given in formula (3). As a result, the contribution of the new structures exceeds the contribution of the Hamiltonian of only the left-hand particles by an approximate factor of two, and one cannot exclude the possibility that ΔH_{eff} does indeed predominate. The absolute value of the matrix element is also of reasonable order of magnitude.

We note that the analog of the term (1) in the Hamiltonian with change of charm vanishes in the $SU(3)$-symmetry limit. If the observed enhancement of the nonleptonic K-meson decays is connected with the operator (1), then there are no grounds for expecting an analogous enhancement of the nonleptonic decays with change of charm.

The authors are grateful to B.L. Ioffe, L.B. Okun', and M.V. Terent'ev for useful discussions.

[1]M.K. Gaillard and B.W. Lee, Phys. Rev. Lett., **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett., **52B**, 351 (1974).

[2]H. Leutwyler, Phys. Lett., 45 (1974); Nucl. Phys. **B76**, 413 (1974); M. Gell-Mann, Report to the New Orleans Conference (1975).