Supersymmetrical renormalizable theory of massive vector non-Abelian field

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A supersymmetrical model is considered, which is invariant also with respect to global $U(1)\otimes SU(n)$ transformations. The model describes renormalizable interactions of scalar, spinor, and vector fields, the masses of all fields being the same.

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A supersymmetrical model describing the interaction of a Dirac spinor, Hermitian scalar, and Hermitian vector fields of equal (nonzero) mass was considered in[1]. In this paper there is constructed an analogous renormalizable model that contains non-Abelian fields. To make the notation more compact, we use the method of superfields[2] as applied to mirror-symmetry superalgebra,[3] with respect to which the considered models are invariant.

To construct a superfield theory of a massive vector field, one can choose as the argument of the Lagrangian the Hermitian scalar superfield

$$\hat{\Psi} = \hat{\Psi}(x, \bar{\eta}, \eta) = \hat{\Psi}_+ + \hat{\Psi}_- - \Psi_1, \tag{1a}$$

with

$$\bar{D}\hat{\Psi}_+ = D\hat{\Psi}_- = 0,$$

$$(\hat{\Psi}_+)^+ = \hat{\Psi}_-, \quad \bar{D} = \bar{D}\bar{s}, \quad D = \overset{+}{s} D, \tag{1b}$$

$$\bar{\eta} = \bar{\eta}\overset{+}{s}, \quad \eta = \bar{s}\eta, \quad \overset{\pm}{s} = \frac{1}{2}(1 \pm \gamma_5), \tag{1c}$$

where $\bar{\eta}$ and η are anticommuting Dirac spinor coordinates, $\bar{D}$ and D are covariant derivatives with respect to these coordinates, and $\hat{\Psi} = \Psi I + \Psi^a \tau^a$. We write down the Lagrangian in the form

$$L = \int d^4x [L_1(x) + L_2(x)], \tag{2}$$

$$L_1 \sim \frac{1}{n\lambda^2} \mathrm{Sp} \int d^2\bar{\eta}\, d^2\eta \{[\bar{D}\bar{\gamma}_\mu (e^{-\lambda\hat{\Psi}} D e^{\lambda\hat{\Psi}})]^2 + \text{H. c.}\}, \tag{3}$$

$$L_2 \sim \frac{1}{n\lambda^2} \mathrm{Sp} \int d^2\bar{\eta}\, d^2\eta \{ e^{\lambda\hat{\Psi}} - \lambda\hat{\Psi} - 1\}, \tag{4}$$

where $\overset{\pm}{\gamma}_\mu = \overset{\pm}{s}\gamma_\mu$. The Lagrangian L_1 describes the interaction of massless fields,[4] and the addition of L_2 leads to the appearance of an equal mass in all fields.

We shall show that the theory with the Lagrangian (2) is renormalizable on the mass shell. To this end, we make the change of variables

$$e^{\lambda\hat{\Psi}} = (1 + \lambda\hat{\Phi}_-) e^{\lambda\hat{\Phi}_1} (1 + \lambda\hat{\Phi}_+), \tag{5}$$

which satisfies the requirements of the equivalence theorem. Recognizing that

$$\int d^4x \int d^2\bar{\eta}\, d^2\eta\, f(\Phi_\pm) = 0, \quad \mathrm{Sp}\ln(\hat{R}\hat{Q}) = \mathrm{Sp}\ln\hat{R} + \mathrm{Sp}\ln\hat{Q},$$

and taking (1b) into account, we obtain

$$L_1 \sim \frac{1}{n\lambda^2} \mathrm{Sp} \int d^2\bar{\eta}\, d^2\eta \{[\bar{D}\bar{\gamma}_\mu (e^{-\lambda\hat{\Phi}_1} D e^{\lambda\hat{\Phi}_1})]^2 + \text{H. c.}\}, \tag{6}$$

$$L_2 \sim \frac{1}{n\lambda^2} \mathrm{Sp} \int d^2\bar{\eta}\, d^2\eta \{ (1 + \lambda\hat{\Phi}_-) e^{\lambda\hat{\Phi}_1} (1 + \lambda\hat{\Phi}_+) - \lambda\hat{\Phi}_1 - 1\}. \tag{7}$$

Since $(\hat{\Phi}_1)^3 = 0$, expressions (6) and (7) are polynomial in the fields. After integrating with respect to $\bar{\eta}$ and η and eliminating the intermediate fields we have (the mass and the coupling constant are set equal to unity):

$$\begin{aligned} L(x) = \frac{1}{n}\mathrm{Sp}\Big\{ & \frac{1}{2}(\partial_\mu\hat{A})^2 + \frac{1}{2}(\partial_\mu\hat{B})^2 - \frac{1}{8}[2\hat{A} + (\hat{A} + i\hat{B})(\hat{A} - i\hat{B})]^2 \\ & - \frac{1}{4}(\partial_\mu\hat{C}_\nu - \partial_\nu\hat{C}_\mu - 2i\check{C}_\mu\check{C}_\nu)^2 + \frac{1}{2}\hat{C}_\mu^2(1 + \hat{A} + i\hat{B})(1 + \hat{A} - i\hat{B}) \\ & - \frac{1}{2}(1 + \hat{A} + i\hat{B}) i\overleftrightarrow{\partial}_\mu (1 + \hat{A} - i\hat{B})\hat{C}_\mu + \frac{i}{2}\bar{\hat{\Psi}}\gamma_\mu\overleftrightarrow{\partial}_\mu\hat{\Psi} \\ & - \bar{\hat{\Psi}}\bar{s}\hat{\Psi}(1 + \hat{A} + i\hat{B}) - \bar{\hat{\Psi}}\overset{+}{s}\hat{\Psi}(1 + \hat{A} - i\hat{B}) + \bar{\hat{\Psi}}\bar{\gamma}_\mu\hat{C}_\mu\hat{\Psi} + 2\bar{\check{\Psi}}\overset{+}{\gamma}_\mu\check{C}_\mu\check{\Psi}\Big\}, \end{aligned} \tag{8}$$

where $\check{C}_\nu = C_\nu^a\tau^a$. Since $\hat{C}_\nu$ is the transverse part of the vector field $(\partial_\nu\hat{C}_\nu \equiv 0)$, the Lagrangian (8) is renormalizable. We note that in the Abelian case the theory with the Lagrangian (8) coincides (on the mass shell) with the theory investigated in[1].

The model considered above can be obtained by spontaneous breaking of gauge supersymmetry. To this end it is necessary to choose expression (3) for L_1 and express L_2 in the form

$$L_2 \sim \frac{1}{n\lambda^2} \mathrm{Sp} \int d^2\bar{\eta}\, d^2\eta \{ \lambda\hat{\Phi}_- e^{\lambda\hat{\Psi}} \lambda\hat{\Phi}_+ - \lambda\hat{\Psi}\}. \tag{9}$$

This model describes the interaction of the gauge field $\hat{\Psi}$ with the fields $\hat{\Phi}_+$ and $\hat{\Phi}_-$ and without the last term in (9) the masses of all fields are equal to zero. Addition of the term $-\lambda\hat{\Psi}$ leads to the appearance of a negative mass squared in the scalar fields. Spontaneous breaking of the gauge supersymmetry, choice of the supergauge $\hat{\Psi}_+ = \hat{\Psi}_- = 0$, and the substitution $\hat{\Psi}_1 = \hat{\Phi}_1$ transforms expression (3) into (6) and (9) into (7). We emphasize that all the scalar, spinor, and vector fields acquire the same mass, with the exception of the field $\hat{B}$ in (8), which is connected with the longitudinal part of the vector field.

The Lagrangian (8) contains besides the non-Abelian fields also Abelian fields. The reason is that the transformation (5), and also the gauge transformation of the fields $\hat{\Phi}_+$ and $\hat{\Phi}_-$ in (9) intermix the singlet and adjoint representations of the $SU(n)$ group. We note also that the use of mirror-symmetry superalgebra has led, as before,[1,3] to a vector-axial character of the spinor fields in (8), i.e., to violation of the P-invariance of the theory.

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