Spontaneous breaking of isotopic symmetry in $K \to 2\pi$ decays

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It is shown that allowance for spontaneous isospin nonconservation leads to weak transitions with $\Delta T = 3/2$ and explains the observed ratio of the widths of the decays $K^+ \to \pi^+\pi^0$ and $K_1^0 \to \pi^+\pi^-$.

It has become possible recently, within the framework of the technique of generalized Ward identities,[1,2] to estimate the influence of direct (non-electromagnetic) breaking of isotopic symmetry on the intramultiplet splitting of the meson masses,[2] on the Cabibbo angle,[3] and others (e.g., the rate of the $\eta \to 3\pi$ decay[4]). The results are in good agreement with experiment.

It is shown below that a detailed and model independent allowance for the isotopic asymmetry of the vacuum is capable of explaining also the observed deviation from the $\Delta T = \frac{1}{2}$ rule in $K \to 2\pi$ decays.

It is assumed that the total Hamiltonian is of the form

$$H = H_0 + h + H_W, \tag{1}$$

where H_0 is $SU(3)\otimes SU(3)$ invariant, h is the breaking and is proportional to local scalar fields s_i $(i = 0, \ldots, 8)$ that transform together with the p_i nonet in accord with the representation $(3, \bar{3}) + (\bar{3}, 3)$, and H_W is the effective weak-interaction Hamiltonian.

We assume that the amplitudes of the $K \to 2\pi$ decays are dominated by a pole (tadpole) diagram (see the figure) (S and W denote renormalized vertices induced by the terms $H_0 + h$ and H_W, respectively). This mechanism explains in natural fashion the strong dependence of the $K \to 2\pi$ amplitudes on the external momentum, a dependence that arises in current algebra and in fact follows from the latter.[5] We make mention here also of the success of the tadpole model in the description of nonleptonic decays of kaons and hyperons, in the calculation of the $K_1^0 - K_2^0$ mass difference, etc.[6,7] As shown by direct calculations, the contributions (with $\Delta T = \frac{1}{2}$) of other diagrams to the $K_1^0 \to 2\pi$ decays amount to about 20% in the current-current model.[8]

Let us now determine more accurately the structure of the term h in (1). It is of the form

$$h = (c_0 s_0 + c_8 s_8) + c_3 s_3, \tag{2}$$

where the first term corresponds to breaking of chiral $SU(2)\otimes SU(2)$ symmetry, and the second to non-electromagnetic breaking of isosymmetry, with the vacuum expectation values of the fields being $\langle 0 | S_i | 0 \rangle \equiv \lambda_i \neq 0 (i = 0, 3, 8)$.

The four-point functions $g_{K^+\pi^+\pi^0 K_1^0}$ and $g_{K_1^0\pi^+\pi^- K_1^0}$ can be connected with the three-point functions with the aid of the Ward identities[2] that follow from the broken $SU(3)\otimes SU(3)$ symmetry (when varying the functional $A(\lambda) = Z - \int d^4x c_i(x)\lambda_i(x)$ with respect to the quantities λ_i; Z is the generating functional for the Green's functions):

$$g_{K^+\pi^+\pi^0K^0} = \frac{\sqrt{2}}{F_\pi}\left[\frac{\sqrt{2}a_{03} + a_{83}}{\sqrt{3}} g_{K^+K_1^0\delta^-} - \frac{i}{2}\left(g_{K^+\pi^0\kappa^-} - g_{K_1^0\pi^0K_1^0}\right)\right], \tag{3}$$

$$g_{K_1^0\pi^+\pi^-K_1^0} = \frac{\sqrt{2}}{F_\pi}\left[\frac{1}{\sqrt{3}}\left(\sqrt{2}\, g_{K_1^0K_1^0 0_s} + g_{K_1^0K_1^0 8_s}\right) + i g_{\pi^- K_1^0 \kappa^+}\right], \tag{4}$$

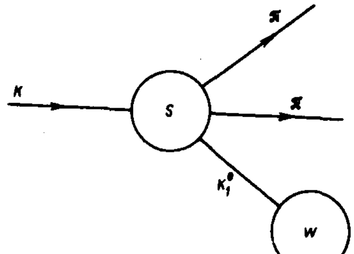
where a_{ij} is the matrix describing the $\pi^0\eta\eta'$ mixing in the pseudoscalar nonet: $\pi^0 = a_{33}p_3 + a_{83}p_8 + a_{03}p_0$: $F \equiv \sqrt{2/3}(\sqrt{2}\lambda_0 + \lambda_8) \approx 0.96 m_{\pi^+}$ is the π^+-meson decay constant; 0_s and 8_s are scalar unitary and isotopic singlets, while δ and κ represent the 0^+ isotriplet and isodoublet, respectively. In the scheme under consideration, the κ-meson mass can be calculated and is equal to $\kappa = (fK - \pi)/(f-1)$,[3] where $f = F_{K^+}/F_{\pi^+}$, while $F_{K^+} \equiv \sqrt{2}(\sqrt{2/3}\lambda_0 - (\lambda_8/2\sqrt{3}) + (\lambda_3/2))$ is the K^+-meson coupling constant (the labels of the particles here and below designate the squares of their masses), while δ can be identified with the experimentally observed $\delta(970)$ 0^+ isotriplet,[9] and the three-point functions in (3) and (4) can be expressed in terms of the $0^\pm$ meson masses and their decay constants. The mixing angles a_{ij} can be obtained from the Ward identities for the neutral channels and from the normalization condition $a_{33}^2 + a_{83}^2 + a_{03}^2 = 1$. They are equal to (accurate to $(\Delta K/K)^2$, where ΔK is the "tadpole" mass splitting in the K-meson isodoublet)

$$a_{33} = 1, \quad a_{83} = 0, \quad 2\sqrt{(2/3)}\, a_{03} = -\frac{\Delta K(\delta - \pi)}{(\delta - K)(K - \pi)}. \tag{5}$$

We obtain finally for the amplitude A_+ of the $K^+ \to \pi^+\pi^0$ decay

$$A_+ = -i\frac{\langle 0 | H_W | K_1^0\rangle}{2K_1^0 F_K + F_{\pi^+}}\Delta K\left[\frac{f}{f-1}\frac{\delta - \pi}{\delta - K} + \frac{f}{(f-1)^2}\frac{K - \pi}{\delta - K} + \frac{\delta - \pi}{K - \pi}\right], \tag{6}$$

and the amplitude A_0 of the $K_1^0 \to \pi^+\pi^-$ decay is of the form



$$A_0 = -\frac{\langle 0 | H_W | K_1^0 \rangle}{K_1^0 F_\pi + F_{K^0}} \left[\kappa^+ - \pi^+ + \frac{4 0_s - 8_s - 3K^0}{3} \right] ,$$

$$F_{K^0} = F_{K^+} - \lambda_3. \tag{7}$$

To calculate the tadpole mass difference ΔK we can use the modified Dashen theorem $f^2(\Delta K)_{em} = (\Delta\pi)_{em} \approx (\Delta\pi)_{exp}$:

$$\Delta K = (\Delta K)_{exp} - (\Delta K)_{em} = 0.0048 \text{ GeV}^2$$

The experimental uncertainty in the masses of the 8_S and 0_S particles does not make it possible to calculate the amplitude A_0 with the same degree of accuracy as A_+. For an approximate estimate of the mass of the 8_s particle we use the Gell-Mann–Okubo formula $3 \cdot 8_S = 4\kappa - \delta$, and we identify the 0_S particle with the broad ϵ resonance of mass ~600–700 MeV.[9]

The ratio R of the amplitudes A_+ and A_0 is already independent of the detailed structure of H_W. For values of f in the interval 1.22 to 1.30 and for an ϵ-meson mass 600 to 700 MeV we have $R = \frac{1}{20}$ to $\frac{1}{30}$, which can be regarded as fairly close to the experimental $R = 0.044$.

We have not considered here the breaking of isotopic symmetry by electromagnetic interactions proper. Their contribution of R was found in a number of studies to be small, on the order of $[(\pi^+ - \pi^0)/K][(\Delta K)_{em}/K]$, making the hypothesis of spontaneous nonconservation of isotopic spin more appropriate and attractive.

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