Topologically nontrivial particles in quantum field theory

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We establish the existence of unusual ("topologically nontrivial") particles in two-dimensional models in a situation wherein there are no solitons in the corresponding classical problem (in particular, for the $\lambda\phi^4$ and $g\bar{\psi}\psi\phi$ interactions at large λ and small g).

It has been recently ascertained that the particle-like solutions of the classical equations can be correlated with particles of the corresponding quantum problem.[1-3] In some cases, topological considerations make it possible to establish that the phase space of the classical problem is not connective; this fact is used to prove the existence of particle-like solutions and the quantum particles corresponding to them (see, e.g., [4]). If a particle-like solution belongs not to that phase-space component in which the classical vacuum lies (the state with lowest energy), then this solution can be naturally called topologically nontrivial. We shall also call the corresponding quantum particle topologically nontrivial; simple considerations show that topologically nontrivial particles are stable. We show in this article that topological considerations can be used to prove the existence of unusual ("topologically nontrivial") particles also in the case when there exist no particle-like solutions.

Let us consider first a two-dimensional model described by the Hamiltonian $H = H_0 + :\ \lambda \int P(\phi(x))dx:$, where H_0 is the Hamiltonian of the free scalar field with mass m, and $P(\phi)$ is an even polynomial. If the minimum of the polynomial $Q(\phi) = (m^2\phi^3/2) + \lambda P(\phi)$ is reached not at the zero point, then we encounter, even in the classical theory, violation of the symmetry $\phi \to -\phi$; this situation obtains also in the quantum problem. The classical equations of motion have in this case soliton with finite energy; as $x \to \infty$ and $x \to -\infty$ these solutions go over into the classical vacua ϕ_+ and ϕ_- (by classical vacuum we mean the number ϕ at which the minimum of the polynomial $Q(\phi)$ is reached). Consider the quantum particle corresponding to the soliton (extremon, in the terminology of [2]). Let $\psi = \psi(f)$ be the state vector of this particle with wave function f. It is easily seen that for any local (or quasilocal) operator we have

$$\lim_{x \to \pm\infty} \langle A e^{iPx} \Psi, e^{iPx} \Psi \rangle = \langle A \Phi_\pm, \Phi_\pm \rangle, \qquad (1)$$

where $\Phi_+ (\Phi_-)$ are the ground states of the quantum problem (physical vacua) corresponding to the classical vacua $\phi_+ (\phi_-)$. (If there are more than two classical vacua in the theory, then the quantum corrections to the energy of the vacua not connected with each other by the symmetry transformation $\phi \to -\phi$ are generally speaking different. This means that not every classical vacuum corresponds to a physical vacuum; it follows therefore from (1), in particular, that not every soliton has a corresponding quantum particle).

We turn now to the situation where the symmetry $\phi \to -\phi$ is broken in the theory described by the Hamiltonian H considered above, but this breaking is not necessarily connected with violation of symmetry in the corresponding classical theory. (It is known, for example, the breaking of this symmetry takes place in the case of strong coupling, i. e., when λ is large enough.[5]) This means that there are in the theory two physical vacua, Φ_+ and Φ_-, which go over into each other under the symmetry $\phi \to -\phi$ (we assume for the sake of argument that there are no other physical vacua). We shall show that in the case considered by us there exist states having finite energy and satisfying the condition (1) (the energy is reckoned, as always, from the ground state). It is natural to call these states topologically nontrivial. (A state should be taken here, strictly speaking, to mean a positive functional on the algebra of the observables. We can use, for example, the L-functional formalism proposed in [6]; then the state described by the L functional, $L(\alpha^*, \alpha)$ should be called topologically nontrivial if

$$\lim_{x \to \pm\infty} L(\alpha_x^*, \alpha_x) = L_\pm(\alpha^*, \alpha),$$

where α_x is the function obtained from the function α by a shift through x, while L_+ and L_- are functionals constructed on different ground states.) The statement that topologically nontrivial states exist leads to the statement that there exist topologically nontrivial particles (at any rate if, as customariy assumed, the *in*-states form a complete system, then there exist topologically nontrivial particles, inasmuch as a topologically nontrivial state cannot be constructed from topologically trivial particles).

To prove the existence of a topologically nontrivial state let us consider the canonical transformation σ defined by $\tilde{a}(x) = e^{i\beta(x)}a(x)$ and $\tilde{a}^+(x) = e^{-i\beta(x)}a^+(x)$, where $\beta(x)$ is a real infinitely differentiable function equal to zero at sufficiently large positive x and equal to π at negative x having a sufficiently large modulus (here $a(x) = 1/\sqrt{2\pi} \int e^{-ikx}a(k)dk$, where $a^+(k)$, and $a(k)$ are the operators for the creation and annihilation of bare particles). The state Ψ obtained from the physical vacuum Φ_+ with the aid of the canonical transformation σ is topologically nontrivial. In fact, the condition (1) can be easily verifed; it is necessary to show only that the energy $\mathcal{E}(\psi)$ of the state Ψ is finite. It is easily seen that $\mathcal{E}(\Psi) = \langle\Psi_+ | H - \tilde{H} | \rangle\Phi_+$, where $\tilde{H}$ is the Hamiltonian obtained from H with the aid of the canonical transformation σ. Using this, we can express $\mathcal{E}(\Psi)$ in terms of the function $\lambda_{m,n}$ obtained from the truncated vacuum mean values $\langle\Phi_+ | a^+(k_1) \ldots a^*(k_m)a(p_1) \cdots a(p_n) | \Phi_+\rangle^T$ by separating the δ functions $\delta(k_1 + \cdots + k_m - p_1 - \cdots - p_n)$. The functions $\nu_{m,n}$ are smooth functions of the momenta; the ultraviolet asymptotic form of these functions can be investigated by perturbation theory, recognizing that in the models under consideration, at large values of the momenta, the effective coupling constant is small. These considerations enable us to confirm, by straightforward but cumbersome calculation, that $\mathcal{E}(\Psi) < \infty$.

In conclusion, let us discuss a model describing massless fermions and massive bosons with an interaction Hamiltonian $g \int \bar{\psi}\psi\phi\, dx$. Iv. Tyutin and E. S. Fradkin have shown that in spontaneous breaking of the symmetry $\phi \to -\phi$, $\psi \to \gamma_5\psi$ takes place in this model at sufficiently small g, and the fermions acquire a mass as a result. Within the framework of the approximation used by them, it can be shown that topologically nontrivial particles exist in this model, too.

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