Spin-flip dynamics and inclusive processes

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A connection between the sign of the imaginary part of the amplitude with spin flip in elastic πN scattering at small momentum transfers and the ratio of the cross sections of the inclusive processes of the type $\pi N \rightarrow NX$ and $\pi N \rightarrow \Delta X$ is established on the basis of the unitarity conditions in the s-channel.

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The main results of experimental investigations of the parameter R and of the polarization πp scattering in the range 6–40 GeV/c[1] are the following two facts:

1. The dynamic spin-flip effect is preserved up to 40 GeV/c, and consequently the "s-channel" amplitude with helicity flip (AHF)$^{1)}$, f_{+-}, apparently contains a P-pole.

2. The sign of Imf_{+-} in the region of small momentum transfers ($q < 0.5$ GeV/c) at 6 GeV/c is positive in $\pi^- p$ scattering and negative in $\pi^+ p$ scattering (in this region, the AHF is determined mainly by the ρ pole). At 40 GeV/c, Imf_{+-} in $\pi^- p$ is negative and small: Imf_{+-}/$q\,$Im$f_{++} \approx 0.1$ (GeV/c)$^{-1}$.

We present below an illustrative AHF model, in which a connection is established, on the basis of the unitarity conditions in the s channel, between the sign of the imaginary part of the AHF at small q and the ratio of the cross sections of the inclusive processes of the type $\pi N \rightarrow NX$(i) and $\pi N \rightarrow \Delta X$(ii). Im$f_{+-}$ turns out to be positive or negative, depending on whether the process (i) or (ii) predominates in the direct channel at the given energy.

We start from the πN-scattering matrix in the c.m.s., $M = M_0 + iM_1 \vec{\sigma} \cdot \mathbf{n}$ in the impact-parameter approximation.$^{1)}$ As is well known, M_0 and M_1 can be expressed in terms of the partial amplitudes $F(+b)$ of particles having an impact parameter b and passing in the (X, Y) plane "to the right" ($b_y = +b$; $b_z = 0$) and "to the left" ($b_y = -b$; $b_z = 0$) of the center of the nucleon polarized along the Z axis (the pion momentum $\mathbf{p}$ is directed along the X axis)

$$M_n = \pi \int \{ F(+b) + F(-b) \} J_n(q^b)\, b\, db, \quad n = 1, 2 \tag{1}$$

$N\uparrow \to N\pi$			$N\uparrow \to \Delta\Pi$		
s_z	L_z	probability	s_z	L_z	probability
$-1/2$	-1	$2/3$	$+3/2$	-1	$3/6$
			$+1/2$	0	$2/6$
$+1/2$	0	$1/3$	$-1/2$	$+1$	$1/6$

According to the optical theorem, the functions $\mathrm{Im}F(+b) = \Sigma\, \vec{\sigma}_{if}(+b)$ can be determined as the partial total cross sections referred to the element of the plane normal to $\mathbf{p}$. At energies $\gtrsim$GeV we have $\sigma_{el} \ll \sigma_t$ and in this case the unitarity conditions yield

$$A_{\pm} \doteq \mathrm{Im}F(\pm b) \cong \sum_{f \neq i} \sigma_{if}(\pm b) + O(1/2 \sum_{f \neq i} \sigma_{if}(\pm b))^2. \quad (2)$$

How can the inequality $A_+ \neq A_-$ arise in the interaction between an incident particle with an axially symmetrical system such as a polarized nucleon? Let us examine the processes $\pi N\uparrow \to NX$ and $\pi N\uparrow \to \Delta X$ on a polarized nucleon from the point of view of the one-pion reggeized exchange (OPER).[3] By virtue of the anomalous parity of the pion trajectory, the baryon vertices of $N\uparrow \to N\pi$ and $N\uparrow \to \Delta\pi$ have an orbital angular momentum, and the possible values of the projections L_z and s_z (the spin projection of the final baryon) are realized with the probabilities given in the table. The oriented orbital angular momentum can be set in correspondence with an average polar momentum $\dot{\mathbf{p}}_x = \langle \mathbf{L}_z \mathbf{r}_{+y}/\mathbf{r}^2 \rangle$, which lies in a plane normal to z in a direction tangential to the orbit. Thus, in the case $\pi N\uparrow \to NX$, when passing "to the left" $(-b)$, $\dot{\mathbf{p}}_x$ is opposite to the momentum of the incident particle, and when passing "to the right" $(+b)$, the incident particle "overtakes" with the virtual particle. The opposite situation occurs in the case $\pi N\uparrow \to \Delta X$. The reactions $\pi N \to NX$ and $\pi N \to \Delta X$ can be regarded as "stripping" of the pion cloud, and the cross sections corresponding to different values of L_z enter additively, by virtue of the orthogonality of the spin functions of the final baryon. We write down these considerations in the form of the conditions

$$A_{\pm} = \sigma_{in}(b^2) + \sigma^{\pm}_{\Delta X}(b^2) + \sigma^{\mp}_{NX}(b^2), \quad (3)$$

where $\sigma^{\pm}$ will be taken to mean the contribution of the interactions with "rotating" particles having a component $\mathring{\mathbf{P}}_x$ directed opposite to the incident particle $(\mathring{\mathbf{P}}_x < 0)$ or catching up with it $(\mathring{\mathbf{P}}_x > 0)$. We shall show that when account is taken of the strong departure from the mass shell, the only processes realized for arbitrary boson exchange are those corresponding to "opposing" kinematics, i.e., that $\sigma^- = 0$.

Let us consider the processes $\pi N \to B_f X$ in the nucleon rest system, (the lab system) for "opposing" (Fig. a) or "overtaking" (Fig. b) kinematics. A pion with momentum $\mathbf{p}$ interacts with a virtual particle at the vertex (ii). As a result of momentum conservation at the vertex (i) we have $\mathbf{p}_f = -\dot{\mathbf{p}}$.

The condition for the strong coupling of the virtual particle with the nucleon is $M_f \gtrsim M_N$. By definition, in the case of opposing kinematics the emission angle of B_f in the lab is $\theta_f^+ < \pi/2$, and in the case of overtaking kinematics $\theta_f^- > \pi/2$. But when $\mathbf{p}$ greatly exceeds the masses of the interactive particles we have

$$\cos\theta_f = [(\mathbf{p}_f^2 + M_f^2)^{1/2} - M_N]\,|\mathbf{p}_f|^{-1}. \quad (4)$$

It is seen from (6) that at $M_f \gtrsim M_N$ the angles $\theta_f > \pi/2$ are forbidden, from which it follows that $\sigma^- = 0$. This condition together with the condition (3) justifies the statements made in the introductory part. We note that in the case of baryon exchange we have $\sigma^- \neq 0$, since M_F can be $< M_N$. This case, however, corresponds to an entirely different region of kinematic variables (small u and backward scattering).

We shall show in conclusion that the presented illustrative arguments conform to a more rigorous approach. We confine ourselves to the case $\pi N\uparrow \to NX$ and verify that at small values of q we have

$$\mathrm{Im}\, f_{+-} = -\pi \int \{\sigma_{NX}(+b) - \sigma_{NX}(-b)\} J_1(qb)\, b\, db > 0. \quad (5)$$

in the one-pion exchange model we have

$$M(\pi N\uparrow \to Nx) = i(\sigma^t_{\pi\pi})^{1/2} F(k^2)\, \bar{\chi}_f \vec{\sigma}\mathbf{k}\, \chi_i\uparrow,$$

where $\mathbf{k}$ is the momentum transferred to the baryon vertex, $F(k^2\ldots)$ is a positive-definite form factor, and $\chi_i\uparrow$ is a spin function of the polarized nucleon. The cross sections $\sigma(+b)$ are defined as the squares of the moduli of the Fourier transforms

$$F(\mathbf{b}) = \langle \bar{\chi}_f | \int \exp(-i\mathbf{k}\mathbf{b}) F(k^2)\, \mathbf{k}\vec{\sigma}\, d\mathbf{k}_\perp | \chi_i^\uparrow \rangle (\sigma_t^{\pi\pi})^{1/2}, \quad (6)$$

taken at $b_y = \pm b$ and $b_z = 0$. Integration over the azimuthal angle yields

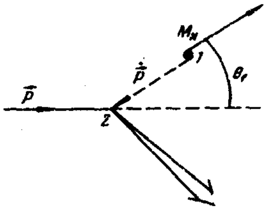
$$F(\pm b) = -i\bar{\chi}_f [(f_0 - f_1)\sigma_x + f_1(\sigma_x \mp i\sigma_y)]\, \chi_i\uparrow (\sigma_t^{\pi\pi})^{1/2},$$

$$f_n = {}^1\!/_2 \int F(k^2) J_n(k_\perp b)\, k_\perp\, dk_\perp^2, \quad n = 0, 1.$$

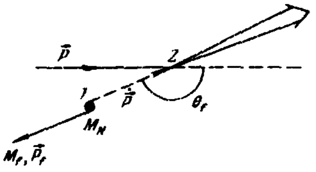
Using the explicit expressions for the spin matrix elements, we have

$$\sigma_{NX}(\pm b) = \sigma^t_{\pi\pi}/4\, (f_0 \mp f_1)^2. \quad (7)$$

A numerical estimate shows that f_0 and f_1 at $b \lesssim 1/m_\pi$ are positive definite functions, with $f_0 > f_1$. Thus, $\sigma_{NX}(+b) < \sigma_{NX}(-b)$. The condition $b \lesssim 1/m_\pi$ is equivalent to the strong-coupling condition and, as seen, leads in the re-



Opposing kinematics



Opposing kinematics

action $\pi N\uparrow \to NX$ to "one-sidedness" of the polarized nucleon of the proper sign.

Experiments in the $\gtrsim$GeV region confirm the conclusions that follow from the model, since the production of N is definitely predominant in $\pi^- p$ collisions and that of Δ in $\pi^+ p$ collisions.[4] At high energies there is no pronounced dominance of N or Δ. However, if it is assumed that in the scaling region the ratios of the cross sections of various inclusive processes do not depend on the energy, then the presence of a small negative contribution of the P pole to the AHF favors the predominance of Δ states at large s.

The author has formulated the main idea of the dynamic AHF model in earlier papers,[5] but in the estimate of the interaction under the conditions of "overtaking" kinematics no account was taken of the departure of the "rotating" particles from the mass shell. Therefore, the conclusion arrived at in these papers, that there is no P pole in the AHF, is incorrect.

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1) At large s and small q we have $f_{++} = G_{++} = M_0$ and $-f_{+-} = G_{+-} = M_1$, where G_{++} and G_{+-} are the Treiman and Wick "s-channel" helicity amplitudes.[2]

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[5] I. I. Levintov, Dynamic Model of Amplitude with Spin Flip, ITEP. Preprint, 1973, No. 36; ZhETF Pis. Red. **20**, 281 (1974) [JETP Lett. **20**, 125 (1974)].