# Penetration and screening effects in electron bremsstrahlung on ions

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A semiclassical theory of electron bremsstrahlung in an arbitrary central field is developed. In the case of radiation from a "hard" multi-electron ion, the theory yields approximate expressions for the total effective emission and for the spectrum. These expressions demonstrate the transition from radiation from a "pointlike" ion to radiation from a bare nucleus.

*Introduction.* Under conditions of a hot plasma ($T_e$ $\sim 10^2$–$10^3$ eV) containing partially stripped heavy-element ions, appreciable interest attaches to the question of bremsstrahlung of an electron in a field that goes over smoothly from the Coulomb field of the nucleus to the Coulomb field of a "pointlike ion." Although both the screening effect and the effect that can appropriately be called "penetration" have by themselves been considered many times before (see[1–7] and elsewhere), until recently there has been no published unified analysis of these effects with an examination of the entire transition region. (Some of the results of a recent paper[8] that pertain to bremsstrahlung are discussed below). The purpose of the present paper is to fill this gap. This is done below within the framework of the capabilities of the Thomas-Fermi model of the ion.

*Semiclassical approach.* The analysis is based on the semiclassical approach developed by us in bremsstrahlung theory. The gist of this approach is to use the classical bremsstrahlung theory[9] in combination with certain quantum limitations, consisting of the requirement that the quantum uncertainties of the classical quantities essential to the radiation process be relatively small.

The main basis for the use of the semiclassical approach are the symmetry properties of the Coulomb field, which lead to a characteristic insensitivity of the
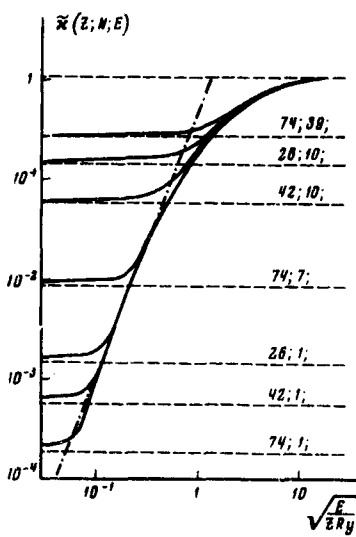
FIG. 1. The ratio $\tilde{\varkappa} \equiv \varkappa(Z; N; E)/\varkappa(Z; 0; 0)$ as a function of $\sqrt{E/ZRy}$; $\varkappa(Z; 0; 0) = 8\pi Z^2 e^6/(3\sqrt{3}\, mc^3\hbar)$. The dashed lines show the asymptotic values of $\varkappa$ (and are marked by the values of $Z$ and $Z-N$). The dash-dot line is a plot of $\tilde{\varkappa} = 0.47(E/ZRy)^{4/3}$.

probabilities of a number of processes to the value of the parameter $\eta = Ze^2/\hbar v$. This insensitivity is most clearly manifest in a dropping out of $\hbar$ from the Rutherford scattering cross section, and in somewhat weaker form in the practical independence of the (dipole) total effective radiation $\varkappa$ of $\eta$. Indeed, in the case of a pure Coulomb field the semiclassical approach makes it possible to duplicate the main results of Sommerfeld's exact quantum-mechanical theory.[10] It becomes clear, in particular, that the classical character of the electron motion ensures also a classical behavior of the bremsstrahlung spectrum.

In the case of a multielectron ion, the field in the transition region $Z - N \lesssim Z_{eff} \lesssim Z$ ($Z$ is the nuclear charge, $N$ is the number of electrons, and $Z_{eff}$ is the "effective" charge that determines the bremsstrahlung intensity) is no longer a Coulomb field, but it can be shown that the well-known condition for quasiclassical motion, $|d\hbar/dr| \ll 1$, is satisfied automatically in the entire region at $Z \gg 1$, so that the semiclassical approach describes the bremsstrahlung correctly in this region, too.

To the contrary, the conditions for the applicability[11] of the Born approximation used in[8] are just as automatically violated in the entire transition region at $Z \gg 1$.

*Total effective radiation.* For an arbitrary central potential $U(r)$, the total effective radiation $\varkappa$ (defined, e.g., in[9]) is given in the framework of the semiclassical approach by

$$\varkappa = \frac{8\pi e^2}{3m^2vc^3} \int_{r_{min}}^{\infty} (dU/dr)^2 \sqrt{1 - U(r)/E}\, r^2 dr, \qquad (1)$$

where $r_{min}$ is the solution of the equation[1])

$$\sqrt{1 - U(r_{min})/E} = \sqrt{3}\, r_{min}/2\hbar \qquad (2)$$

($v$ is the electron velocity, $E = mv^2/2$, $\hbar = \hbar/mv$).

For the case of Coulomb field $U = -Ze^2/r$ we obtain

$$\varkappa = 4\pi Z^2 e^6 [(1+2x)^{3/2} - 1] / [3\sqrt{3}\, mc^3\hbar x\, (1+2x)^{1/2}], \qquad (3)$$

where $x = Ze^2/(mv^2 r_{min})$, and $r_{min}$ is the solution of the corresponding equation (2). Apart from a factor close to unity, Eq. (3) coincides with the result of the exact theory[10] at all values at $\eta$.

For the Thomas-Fermi model of the ion, $U = -(Ze^3/r)\chi(r)$, we have tabulated the results of (1): certain typical curves are shown in Fig. 1, and with additional averaging over the Maxwellian distribution they are shown in Fig. 2.

*Screening and penetration energies.* For an illustrative description of the function $\varkappa(E)$ in the transition region, it is convenient to introduce two characteristic energies that determine the limits of this region, viz., *the penetration energy $E_p$* and the *screening energy $E_s$,* defined by the natural conditions

$$\varkappa(Z; N; E_p) = 2\varkappa(Z-N; 0; 0); \quad \varkappa(Z; N; E_s) = \tfrac{1}{2}\varkappa(Z; 0; \infty). \qquad (4)$$

For the Thomas-Fermi model of the ion we obtain ($\xi \equiv N/Z$; $1 - \xi \ll 1$)

$$E_p \approx 38(1-\xi)^{3/2} Z \text{ eV}; \quad E_s \approx 92Z \text{ eV}. \qquad (5)$$

The *temperatures* $T_p$ and $T_s$ defined in analogy with (4) are

$$T_p \approx 18(1-\xi)^{3/2} Z \text{ eV}; \quad T_s \approx 54Z \text{ eV}. \qquad (6)$$

At $E_p \lesssim E \ll E_s$, the quantity $\varkappa$ varies approximately in proportion to $Z^{2/3}E^{4/3}$ (see Fig. 1).

*Bremsstrahlung spectrum.* With the aid of the semiclassical approach we can find the bremsstrahlung spectrum $d\varkappa(\omega)$ in an arbitrary central field. For the case of a Coulomb field the result takes the form

$$d\varkappa(\omega) = \frac{4\pi^2}{3} \frac{Ze^4v}{mc^3} \left\{ i\nu(1 + 1/\eta') H_{i\nu'}^{(1)}[i\nu(1 + 1/\eta')] H_{i\nu'}^{(1)\prime}[i\nu(1 + 1/\eta')] \right\} d\nu_i . \qquad (7)$$
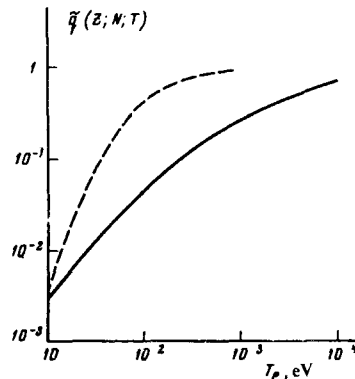


FIG. 2. The ratio $\tilde{q} \equiv \langle v\varkappa(Z; N; E)\rangle / \langle v\varkappa(Z; 0; 0)\rangle$ as a function of $T_e$. Solid curve—present calculation, dashed—from the data of[8], for $Z = 74$ and $Z - N = 1$.
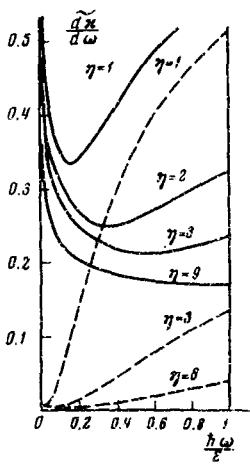
FIG. 3. Reduced bremsstrahlung spectrum $\widetilde{d\varkappa/d\omega} \cong (mv^2 d\varkappa/d\omega)/(2\hbar\varkappa(Z;0;0))$ calculated from formulas (7) and (8). Solid curves—for $Z = 26$ and $Z - N = 10$; dashed—for $Z = 26$ and $Z - N = 1$. The numbers at the curves are the values of the parameter $\eta = \sqrt{Z^2 Ry/E}$.

where $\nu = Ze^2\omega/mv^3$, $\eta' = \gamma\eta$ ($\gamma = 1.78$ is the Euler constant), and $H^{(1)}$ and $H^{(1)'}$ are the Hankel function and its derivative with respect to the argument.

Formula (7) accounts well for the results of the exact theory, [10] with the exception of the limiting case of the vicinity of the short-wave limit at "Born" energies ($\eta \ll 1$), this being due to failure to take into account, in our semiclassical approach, the strong change of the type of trajectory in the radiation act, from linear to parabolic[2], a change typical of this case.

For a Thomas-Fermi ion, by virtue of the aforementioned quasiclassical character of the motion in the transition region and (as can be shown) by virtue of the small radial dimension of the zone responsible for the bremsstrahlung, it is also possible to use approximately formula (7), in which $Z$ is replaced *throughout* by the effective charge $Z_{eff}(v, \omega)$ corresponding to the distance $r_0$ responsible for the radiation at the given frequency: $Z_{eff}(v, \omega) = Z\chi(r_0)$, where $r_0$ is the solution of the equation

$$\overline{\sqrt{1 + 2Ze^2\chi(r_*)/mv^2 r_*}} = 4 r_* \omega / \pi\sqrt{3} v. \quad (8)$$

Some of the results of the calculation by formulas (7)–(8) are shown in Fig. 3.

*Discussion.* Our results can be used to calculate the radiative losses of a plasma and for plasma diagnostic in the ranges of $Z$, $N$, and $T_e$ of practical interest. It

is necessary in this connection to compare our results with the results of[8], which also pertain to a Thomas-Fermi ion, but are based on the Born approximation, which is not valid here. The difference turns out to be quite appreciable, not only numerically (see Fig. 2) but also in the form of the dependence on the parameters $Z$, $N$, and $T$. Thus, the values of $T_p$ and $T_s$ extracted from the results of[8] turn out to be (cf. Eq. (6)):

$$T_p \approx 9.6(1 - \xi)Z^{2/3} \text{ эв}; \quad T_s \approx 7.2 Z^{2/3} \text{ eV}, \quad (9)$$

Which is equivalent to a strong overestimate of $\langle v\varkappa \rangle$ in[8].

[1] We note that in the general case Eq. (2) does not reduce to the customarily employed condition (see, e.g., [12]) $\rho_{min} \sim \lambda$ (where $\rho$ is the impact parameter), a condition valid only in the Born limit.

[2] The possibilities afforded by the semiclassical approach can be extended by a procedure of "symmetrization" with respect to the initial and final states, a procedure extensively used in the theory of Coulomb excitation of nuclei.

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