# Collective modes of a system of electrons on a helium surface

V. B. Shikin

*Institute of Solid State Physics, USSR Academy of Sciences*
(Submitted July 22, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **22**, No. 6, 328–331 (20 September 1975)

The dispersion law for longitudinal oscillations in a system of localized surface electrons is derived. It is shown that the helium-surface deformation accompanying the electron localization has a noticeable effect on the form of the spectrum and leads to the appearance of a threshold frequency.

PACS numbers: 73.20.H

Collective excitations of a Wigner crystal made up of electrons localized on a flat helium surface were investigated by Crandall.[1] It turned out then that in the indicated system there are longitudinal oscillations with a spectrum $\omega(q)$ given by

$$\omega^2 = \frac{2\pi e^2 n_s}{m} q \qquad (1)$$

$\omega$ and $q$ are the frequency and the wave number of the oscillations, $m$ and $e$ are the mass and charge of the electron, $n_s$ is the average density of the surface electrons (usually $n_s \approx 10^8 - 10^{10}$ cm$^{-2}$); The system has also two transverse oscillation modes that have an acoustic character at the long-wave limit.

Actually, however, the liquid–vapor boundary does not remain flat if it carries surface electrons that are pressed towards the surface from the gas phase by the strong electric field $E_\perp$; instead, the surface becomes self-consistently deformed under each of the electrons localized on the surface.[2] The relief of this deformation has large inertia, and in the problem of the spectrum of the electronic oscillations it can be assumed to be static. As a result, the electrons that oscillate about their equilibrium positions are influenced not only by the self-consistent Coulomb forces, but also deformation forces that tend to return each electron to the center of the deformation potential well. In this article we calculate the influence of the deformation forces on the oscillation spectrum of the electrons in helium. The hydrodynamic approximation used below allows us to assess the character of this influence only on the longitudinal mode of the oscillations (1).

The initial system of equation, in the adiabatic approximation,[1)]

$$\ddot{\xi} = -\omega_\sigma^2 \xi + \frac{e}{m} E_x, \qquad \omega_\sigma = \frac{(eE_\perp)^2}{2\pi h \sigma}$$

$$\dot{n} + n_s \frac{\partial \xi}{\partial x} = 0, \quad n << n_s \qquad (2)$$

$$\mathrm{div}\, \mathbf{E} = 4\pi e n \delta(z)$$

consists of the equation of motion for one electron, the continuity equation, and the Poisson equation. Here $\xi(x, t)$ is the amplitude of the oscillations of an individual electron about the equilibrium position and along the interface, $\omega_\sigma$ is the natural frequency of the electron as it moves in the deformation well,[2] $E_\perp$ is the intensity of the clamping electric field, $\sigma$ is the coefficient of surface tension on the vapor–liquid interface, $n(x, t)$ is the alternating increment of the surface density of the electron, the $x$ axis is directed along the helium surface, and the $z$ axis is normal to it. The Poisson equation, in which $\delta(z)$ is a delta function, is three-dimensional and is solved with boundary conditions in terms of $z$; these conditions correspond to the damping of the electric field with increasing distance from the charge surface. Such boundary conditions do not take into account the existence of a metallic substrate, which must be present in the problem of surface electrons, but is located at a macroscopically large depth away from the charged surface. Consequently, the wave solution of the system (2), which is obtained below, has in terms of the wave numbers $q$ an upper bound $q \ll n_s^{1/2}$ and a lower bound $q \gg d^{-1}$.

Assuming

$$\xi = \xi_0 e^{i(qx - \omega t)}, \quad \mathbf{E} = \nabla\phi, \quad \phi = f(z) e^{i(qx - \omega t)}$$

and solving the system (2), we obtain the dispersion law of the longitudinal oscillations of the ensemble of the surface electrons

$$\omega^2 = \omega_\sigma^2 + \omega_q^2, \quad \omega_q^2 = \frac{2\pi e^2 n_s}{m} q \quad . \tag{3}$$

Compared with (1), a threshold frequency $\omega_\sigma$ has appeared here.

Turning on a magnetic field of intensity $H$ in a direction normal to the surface of the helium alters the dispersion law of the longitudinal oscillations in the following manner:

$$(\omega^2 - \omega_\sigma^2 - \omega_q^2)(\omega^2 - \omega_\sigma^2) = (\omega_H^0 \omega)^2, \quad \omega_H^0 = \frac{eH}{mc} \tag{4}$$

where $c$ is the speed of light.

It is meaningful to note that the action of homogeneous fields, namely an alternating electric field parallel to the helium surface, or a constant magnetic field normal to this surface, on the system of surface electron is not very sensitive to the concentration of the surface electrons, since a homogeneous field does not change the relative position of the electrons, and consequently does not change their mutual potential energy. As a result, for example, an alternating electric field along the helium surface, $E_\parallel(t) = E_0 e^{i\omega t}$ causes harmonic oscillations $\xi(t)$ of the electrons about the positions of their equilibrium, without exciting oscillations of the electron density

$$\xi(t) \equiv \frac{e E_\parallel(t)}{m(\omega_\sigma^2 - \omega^2)} \,. \tag{5}$$

An analogous situation is observed for the frequency $\omega_H$ under conditions of cyclotron resonance

$$\omega_H = \left[\omega_\sigma^2 + \frac{1}{2}(\omega_H^0)^2\right]^{1/2} \pm \frac{1}{2}\omega_H^0 \,. \tag{6}$$

The shift of this frequency, in comparison with the value $\omega_H^0$ for one electron, is possible only to the extent that $\omega_\sigma \neq 0$. The onset of deformation forces that lead to the onset of the frequency $\omega_\sigma$ becomes sufficiently probable under conditions when the total energy gain $W_\sigma$ due to the self-consistent bending of the surface under each of the electrons begins to exceed noticeably the temperature[3]

$$W_\sigma \gg T, \quad W_\sigma = \frac{1}{4}\hbar\omega_\sigma \ln Q, \quad Q = \frac{m\omega_\sigma}{\hbar n_s} \gg 1. \tag{7}$$

The inequality (7) depends logarithmically on $n_s$, and this determines the manner in which $n_s$ influences the frequencies $\omega_\sigma$ and $\omega_H$.

The hitherto reported experimental measurements of the cyclotron frequency[4] were made under the following conditions: $H \approx 10^4$ Oe, $T \approx 1.2\,^\circ$K, $n_s \approx 10^8$ to $10^9$ cm$^{-2}$, $E_\perp = 4\pi e n_s \approx 6\times(0.1$ to $1)$ cgs esu, and $\sigma = 0.36$ erg/cm$^2$. As a result we have $\omega_H^0 \sim 10^{11}$ sec$^{-1}$, $\omega_\sigma \sim 10^8 - 10^{10}$ sec$^{-1}$, and $W_\sigma \approx (10^{-2} - 10^{-1})^\circ \ll T$. Thus, the inequality (7) is essentially not satisfied, and therefore the frequency $\omega_\sigma$, which is comparable in order of magnitude with $\omega_H^0$, is in fact not formed, since the deformation localization of the electrons at these temperatures is of low likelihood. As a result, the cyclotron frequency $\omega_H$ should have a value that coincides with $\omega_H^0$, as is indeed observed in experiment.

The author is grateful to L. P. Gor'kov for a discussion of the results.

1) The effective mass $M$ of the deformation "pit" as it moves along the surface is of the order of $M \sim 10^3 m_{He^4}$.[2] The adiabaticity parameter is therefore $\epsilon = m/M \sim 10^{-6} \ll 1$.


1. R. S. Crandall, Phys. Rev. **A8**, 2136 (1973).
2. V. B. Shikin and Yu. P. Monarkha, Zh. Eksp. Teor. Fiz. 65, 751 (1973) [Sov. Phys.-JETP 38, 373 (1974)].
3. Yu. P. Monarkha and V. B. Shikin, *ibid.* **68**, 1423 (1975) [41, No. 4 (1975)].
4. T. R. Brown and C. C. Grimes, Phys. Rev. Lett. **29**, 1233 (1972).