Description of elastic scattering in the *U*-matrix method

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(Submitted July 2, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **22**, No. 6, 347–350 (20 September 1975)

An analysis is presented of elastic pp scattering on the basis of a generalized matrix of the reactions (the U matrix). Good agreement is obtained with the experimental data on $\sigma_{tot}(pp)$ starting with 30 GeV, and for $d\sigma/dt\,(pp)$ for four ISR energies.

PACS numbers: 13.80.D, 11.80.G

1. In [1] we proposed a method of coupling the direct and annihilation reaction channels based on an analogous continuation in the crossing channel of the U matrix connected with the scattering amplitude by the equal-time equation in quantum field theory. [2] This method of connecting the channels retains unitarity on going to the s channel, without any additional assumptions whatever concerning the behavior of the trajectories, or without any limitations on the coupling constant, and makes it possible to examine in the behavior of the hadron-interaction cross sections in a simple and unified manner, all the regimes that are allowed by the unitarity condition.

The Regge asymptotic terms were obtained in [1,3] for the even and odd parts of the U matrix at high s-channel energies

$$U^{\pm}(s,t) = -g^{\pm}(t)\xi^{\pm}(t)s^{\beta^{\pm}(t)}, \tag{1}$$

where $\xi^{\pm}(t)$ are the signature factors, and the leading trajectories are designated by $\beta^{\pm}(t)$. We note that the limitation $\beta(0)\leqslant 1$, which is needed to conserve unitarity in the direct channel in the case when amplitude is analytically continued, is not imposed in the derivation of (1). The s-channel scattering amplitude is determined by solving the relativistic equation of damping theory, [2] which is of the following form in the c. m. s.:

$$F^{\pm}(\mathbf{p},\mathbf{q}) = U^{\pm}(\mathbf{p},\mathbf{q}) + \frac{i\pi q}{4\sqrt{s}}\int d\Omega_{k}\, U^{\pm}(\mathbf{p},\mathbf{k})F^{\pm}(\mathbf{k},\mathbf{q}), \tag{2}$$

where $p=k=q$, and $F^{\pm}(\mathbf{p},\mathbf{q})$ is the scattering amplitude in the invariant normalization. In partial waves, Eq. (2) reduces to algebraic. Going over to the impact-parameter representation, we represent the scattering amplitude in the form

$$F(s,t) = \frac{is}{\pi^2}\int_0^{\infty}\frac{U(b,s)}{1+U(b,s)}J_0(b\sqrt{-t})\,b\,db, \tag{3}$$

where $iU(b,s)$ is determined by the transformation of the Bessel function $U(s,t)$. Assuming the function $g(t)$ to be constant and the trajectory $\beta(t)$ linear, we obtain $U(b,s)=u(s)\exp(-b^2/a)$, where $U(s)=2\pi^2 gi\xi(0)a^{-1}\times(s)s^{\beta(0)-1}$ and $a(s)=4\beta'(0)[\ln s - i(\pi/2)]$. We then obtain for the total cross section $\sigma_{tot}(s)=4\pi\,\mathrm{Re}\{a(s)\times\ln[1+u(s)]\}$, from which we readily see that the asymptotic term in the total cross section is $\sigma_{tot}^{(\infty)}(s)\sim gs^{\beta(0)-1}$ if $\beta(0)\leqslant 1$. When $\beta(0)>1$ we have $\sigma_{tot}^{(\infty)}(s)=16\pi\beta^{1}(0)\times[\beta(0)-1]\ln^2 s$. Let us estimate the integral (3) for $t\neq 0$. We use for this purpose the asymptotic representation of the Bessel function, $J_0(z)\cong(2/\pi z)^{1/2}\,\mathrm{Re}\exp[iz-i(\pi/4)]$. It is then easy to obtain the estimate

$$F(s,t) = is(4/\pi)^2\sqrt[4]{a^2(s)B(s,t)}\exp\left\{-i\frac{\pi}{2}\sqrt{-tB(s,t)}\right\} \times\cos\left[\sqrt{\frac{-t}{4B(s,t)}}\left(\frac{a(s)}{2}-i\pi\beta(0)\right)\right], \tag{4}$$

where $B(s,t)$ is a slowly varying function of s and t, with $B(s,t)\to$ const as $s\to\infty$. We note the appearance of an oscillating factor in the expression for the amplitude, and also the fact that the argument of the cosine function is proportional to $\ln s\sqrt{-t}$ as $s\to\infty$. The ap-

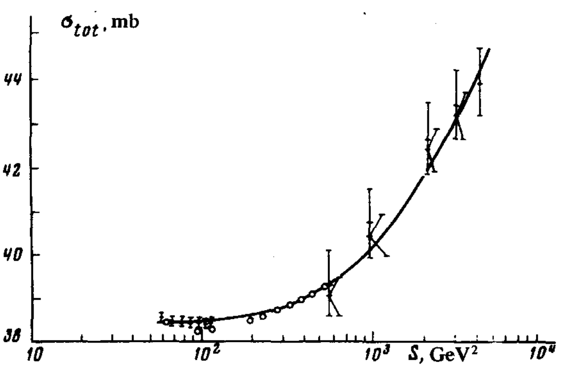

Fig. 1. Total cross section of pp scattering.

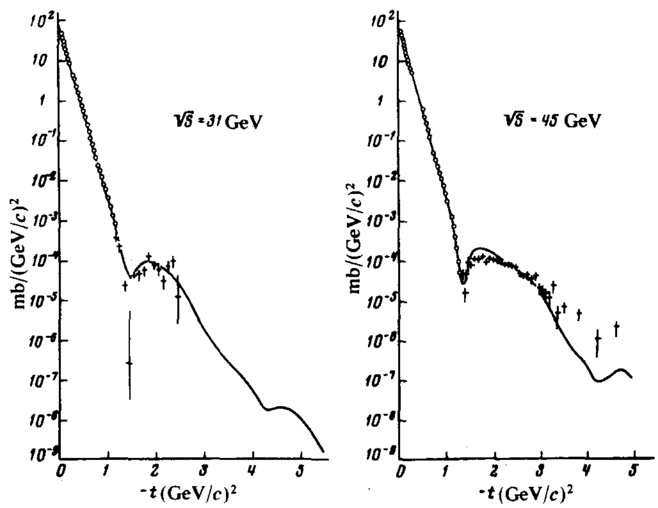

Fig. 2. Differential cross section of elastic pp scattering at ISR energies.

pearance of ln s is due to the fact that $u(s) \sim s^{\beta(0)-1}$, $\beta(0) > 1$, i.e., it is connected with the growth of the total interaction cross section as $s \to \infty$. Thus, the oscillations in the angular distribution of the elastic scattering should become more noticeable with increasing energy, and within the framework of the present approach this is connected with an increase of $\sigma_{tot}(s)$ as $s \to \infty$. It is easy to see that in the case of small momentum transfers the estimate for the amplitude is of the form

$$F(s, t) \sim \exp(a(s)t/2).$$

2. On the basis of the representation (3) for the scattering amplitude, when the function $U^+(s, t)$ is a sum of two terms of the type (1), we have compared the results with experimental data on elastic pp scattering. The functions $g(t)$ and $g_1(t)$ were parameterized in the form $g \exp(bt)$ and $g_1 \exp(b_1 t)$, respectively. As the function $\beta_1(t)$ we chose a linear trajectory passing through the A_2 meson: $\beta_1(0) + m_{A_2}^2 \beta_1'(0) = 2$. The parameter s_0 was introduced for the second pole, and for the first pole $s_0 = 1$ GeV^2. Thus, the reconciliation with the experimental data on $\sigma_{tot}(pp)$ in the IHEP, FNAL, and ISR energy region[4] and on $d\sigma/dt$ in the ISR energy region[5] was effected by choosing eight free parameters:

$$\tilde{g} = 2\pi^2 g,\ \tilde{g}_1 = 2\pi^2 g_1,\ b,\ b_1,\ \beta(0),\ \beta'(0),\ \beta_1'(0).$$

The results of the calculations are shown in Figs. 1 and 2. The numerical values of the parameters are

$$\begin{array}{ll} \tilde{g} = 0.096 \pm 0.008; & \tilde{g}_1 = 57.4 \pm 1.2; \\ b = 2.32 \pm 0.09\,(\mathrm{GeV}/c)^{-2} & b_1 = 2.93 \pm 0.08\,(\mathrm{GeV}/c)^{-2}; \\ \beta(0) = 1.37 \pm 0.01; & \beta'(0) = 0.04 \pm 0.01\,(\mathrm{GeV}/c)^{-2}; \\ \beta_1'(0) = 0.634 \pm 0.001\,(\mathrm{GeV}/c)^{-2}; & s_0 = 14.6 \pm 0.1\ \mathrm{GeV}^2. \end{array} \quad (5)$$

The set of parameters (5) was obtained by fitting formula (3) to the data on $\sigma_{tot}(s)$, starting with $s = 60$ GeV^2 and the data on $d\sigma/dt$ at $\sqrt{s} = 53$ GeV. The values of the differential cross section at energies $\sqrt{s} = 23.5$, 30.7, 44.9, and 62 GeV were then calculated by using the obtained values of the parameters (5). Figure 2 shows the theoretical curves for $d\sigma/dt$ (pp) at $\sqrt{s} = 30.7$ and 45 GeV. The agreement with the experimental data at $\sqrt{s} = 23.5$, 53, and 62 GeV is just as good.[6] The asymptotic term in the total cross section is $\sigma_{tot}(s) = 0.74 \ln^2 s$. With increasing energy, the position of the first minimum shifts towards smaller $|t|$, and the cross section in the region of the maximum that follows increases. Both behaviors agree with experiment. The differential cross section has a second minimum near $-t = 4$ (GeV/c)^2.

The authors are deeply grateful to A. A. Logunov for interest in the work and for valuable advice. The authors are indebted to V. I. Savrin for a discussion.

[1] N. E. Tyurin and O. A. Khrustalev, IFVÉ Preprint 74-119, Serpukhov, 1974.
[2] A. A. Logunov, V. I. Savrin, N. E. Tyurin, and O. A. Khrustalev, Teor. Fiz. 6, 157 (1971).
[3] V. I. Savrin and N. E. Tyurin, *ibid.* 23, 348 (1975).
[4] S. P. Denisov, S. V. Donskov, Yu. P. Gorin, *et al.*, Phys. Lett. **36B** 415 (1971); A. S. Carroll, I. H. Chiang, T. F. Kycia *et al.*, Phys. Rev. Lett. 33, 928 (1974); U. Amaldi, R. Biancasteli, C. Bosio, *et al.*, Phys. Lett. **44B**, 112, (1973). S. R. Amendolia, G. Bellettini, P. Braccini, *et al.* Phys. Lett. **44B**, 119 (1973).
[5] G. Barbiellini, M. Bozzo, P. Darrivlat, *et al.*, Phys. Lett. **39B**, 663 (1972). A. Böhm, M. Bozzo, R. Ellis, *et al.*, Phys. Lett. **49B**, 491 (1974).
[6] V. F. Edneral, S. M. Troshin, N. E. Tyurin, and O. A. Khrustalev, IHEP Preprint 75-83, Serpukhov, 1975.