Metastable states of diffuse neutral sheet and the substorm explosive phase

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We consider the nonlinear theory of explosive instabilities of the diffuse neutral sheet produced as a result of resonant interaction with electrons or ions. It is shown that in sufficiently thin sheets there can occur metastable states relative to the buildup of ion instability. The results can be used to explain theoretically the explosive phase of magnetospheric substorms.

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The release of the energy stored in plasma configurations with self-consistent magnetic fields having diffuse neutral sheets (for example, in the tail of the earth's magnetosphere) are attributed to the development of an explosive mode of plasma instability.[1]

As shown in[2], quasilinear effects have a weak influence on the development of the instability, so that the perturbations grow to amplitudes at which nonlinear effects must be taken into account. The most important nonlinear effect is the qualitative change of the particle trajectories in the magnetic field of a perturbation of finite amplitude in a narrow zone near the neutral sheet, where the plasma is not magnetized and the particles can exchange energy effectively with the perturbation. When a large number of modes with different propagation directions are superimposed in the plane of the neutral sheet, the nonlinear influence of the magnetic field of the perturbations on the particle trajectory can be replaced by the effect of the presence of a constant magnetic-field component vertical to the layer. The solution of the problem of the stability of a neutral sheet with a small vertical component is also of importance when it comes to determining the instant when a neutral line is formed in the strongly compressed tail of the earth's magnetosphere.

If the ratio of the normal magnetic-field component B_z to the unperturbed field B_{0x} is a quantity of higher order of smallness than the employed expansion parameter ρ_i/L (ρ_i is the Larmor radius of the ion and L is the characteristic dimension of the sheet), then we can take the equilibrium state to be the well known Harris configuration[3]

$$B_x = B_0 \,\mathrm{th}(z/L), \qquad n = \frac{n_0}{\mathrm{ch}^2(z/L)}, \tag{1}$$

which corresponds to particle-velocity distribution functions having a constant shift along the y component of the velocity in the entire neutral sheet.

In this case the problem of plasma stability to perturbations with a vector potential

$$A_y(\mathbf{r}, z, t) = A(z) e^{-i\omega t + i\mathbf{k}\mathbf{r}} \tag{2}$$

reduces to the solution of the problem of the frequency eigenvalues for an equation of the Schrödinger type:

$$\frac{d^2 A}{dz^2} - [k^2 + V_0(z) + \sum_{j=i,e} V_j^<(z, \omega, k)]A = 0, \tag{3}$$

where

$$V_0 = -2L^{-2}\,\mathrm{ch}^{-2}(z/L),$$

$$V_j^< = \begin{cases} 0, & |z| > d_j \equiv \sqrt{\rho_j L} \\ \dfrac{2\omega_{pj}^2}{c^2} \displaystyle\int_0^\infty x e^{-x} \sum_{n=-\infty}^{+\infty} J_n'^2(\Lambda\sqrt{x})\, dx \frac{\omega}{\omega - n\Omega_j + i0}, & |z| < d_j, \end{cases}$$

$$\omega_{pj}^2 = \frac{4\pi e_j^2 n_0}{m_j}, \quad \Lambda^2 = \frac{2k^2 T_j}{m_j \Omega_j^2}, \quad \Omega_j = \frac{e_j B_z}{m_j c}, \quad \rho_j^2 = \frac{2m_j c^2 T_j}{e_j^2 B_0^2}$$

J_n' is the derivative of the Bessel function with respect to the argument. We see that the effective potential energy takes the form of a shallow well with a high and narrow potential barrier in its center (Fig. 1), due to the contribution of the particles from the narrow region near the neutral sheet. In the limit of a planar neutral sheet ($B_z \equiv 0$), the summation over "n" in Eq. (3) is easily carried out, and we get

$$V_j^< = -\frac{i\pi^{1/2}\omega}{|k| v_{Tj}} \frac{\omega_{pj}^2}{c^2}. \tag{4}$$

We see that the minimum height of the barrier[1)]

$$V_e^< d_e L < 1, \tag{5}$$

at which the shallow potential well $V_0(z)$ still contains an "energy" level $-k^2 \sim -L^{-2}$, determines the maximum increment of the electronic branch of the instability $\gamma_e \approx k v_{Te}(\rho_e/L)^{3/2}$. When the magnetic-field component normal to the layer is increased under the condition $\Omega_e - \gamma_e$, Cerenkov interaction of the electrons with the perturbation becomes impossible. At the same time,

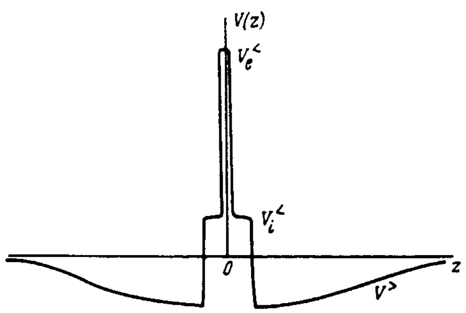

FIG. 1. Form of the effective potential $V_0(z)$.

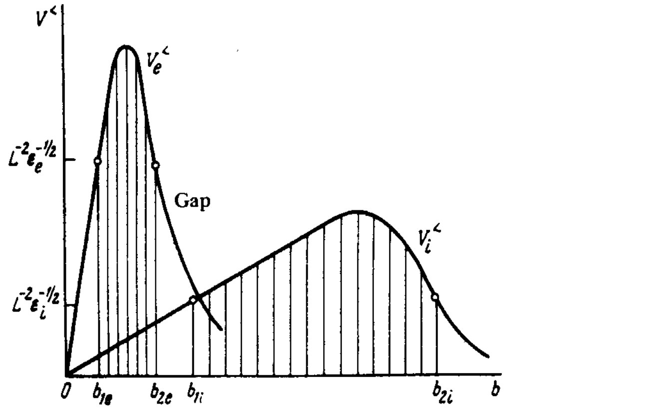

FIG. 2. Dependence of the height of the potential barrier on the normal field component.

the condition (5) for the presence of a negative energy level "$-k^2$" is violated in a wide interval of the values of the normal field component (Fig. 2)

$$\epsilon_e^{5/2}\left(1 + \frac{T_i}{T_e}\right) \approx b_{1e} < b \equiv \frac{B_z}{B_0} < b_{2e} = \epsilon_e^{1/4}\left(1 + \frac{T_i}{T_e}\right)^{-1/2}. \tag{6}$$

Quite analogously, in the interval $b_{1i} < b < b_{2i}$ the level becomes crowded out because of the partial inpenetrability of the potential barrier V_i. If the parameters are such that $b_{1i} > b_{2e}$, then a finite perturbation exists in the "gap" $b_{2e} < b < b_{1i}$ and interacts effectively with the ions; this leads to an instability with a growth rate $\gamma_i \sim (\rho_i/L)^{3/2} v_{Ti}/L$ (Fig. 2).

The complete diagram of the plasma stability in the plane of the parameters (b, ϵ_i) is shown in Fig. 3. The metastable states at $b < b_{2e}$ are unstable to finite-amplitude perturbations that take the parameter b out into the unstable "gap." The presence of such states appears to play a decisive role in processes of rapid spontaneous restructuring of the topology of the earth's magnetosphere tail, which take place during the time of substorms. According to satellite measurements,[4] a decrease of the magnetic-field component normal to the sheet takes place during the stage preparatory to the substorm, and the plasma sheet becomes thinner, so that the system goes over to the metastable state B

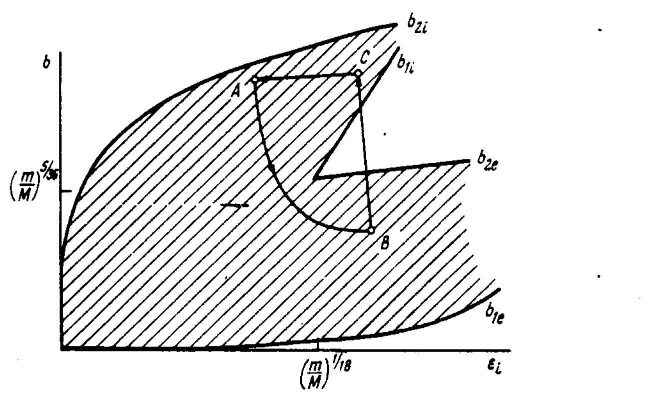

FIG. 3. Stability diagram (the stability region is shown shaded).

under the gap, from which it is capable of going over spontaneously to the state C (Fig. 3). This mechanism of rapid spontaneous restructuring of the magnetosphere tail greatly facilitates the construction of a theory of the explosive phase of substorms on the basis of the explosive instability of the neutral sheet.[5,6]

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[1)]We note that levels with $k^2 \to 0$ vanish only if the barrier is entirely inpenetrable, i.e. $V_e \sim d_e^{-2}$. However, perturbations of such large wavelength cannot develop in a neutral sheet of finite length L.

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