Self-action of electromagnetic waves in a plasma under modulation instability

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It is shown that the development of modulation instability in the skin layer can lead to a bleaching of a dense opaque plasma under the influence of an intense electromagnetic wave. The characteristic parameters of the bleaching are determined.

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We discuss here one of the mechanisms of bleaching an opaque plasma under the influence of an intense electromagnetic wave. The bleaching is due to the formation in the opaque plasma of waveguide channels resulting from the development of modulation instability of the plasma oscillations in the skin layer of the incident wave. In contrast to the known models,[1,2] the proposed mechanism explains a number of experimentally observed singularities of nonlinear bleaching,[3,4] and primarily the short time of establishment of the bleaching, and also the presence of groups of fast particles and the existence of satellites in the spectrum of the radiation that passes through the plasma.

Consider the incidence of a plane monochromatic electromagnetic wave on a weakly-inhomogeneous plasma layer with dielectric constant $\epsilon_0(z)$ and a concentration at the maximum exceeding the critical value, $\epsilon_0(z)=\epsilon_M<0$. We assume that at the initial instant of time the usual "linear" distribution of the electromagnetic-wave field has been established in the plasma, with a maximum field amplitude $E=E_m$ near the reflection point $\epsilon_0=0$. If the condition $E_m^2/16\pi N(T_e+T_i) > \nu/\omega_{pe}$ is satisfied, where ν is the effective frequency of the electron collisions and N, T_e, and T_i are the concentration and temperatures of the electrons and ions, then it becomes possible for a modulation instability to develop[5] and to lead to formation in the plasma of nonlinear distributions of standing plasma oscillations[1)] (in the direction of the electric field $\mathbf{E}$ of the incident wave) and to stratification of the plasma. The resultant plane-layered structure with alternating-sign dielectric constant ($\epsilon>0$ in the antinodes of the plasma-oscillation field) can be transparent to the incident wave, since the layers are perpendicular to its electric field, the distance between them is small in comparison with the length of the electromagnetic wave, and the parameters of the structure vary smoothly along the z axis.

We shall assume that the characteristic scale (period) of the stratification coincides through the instability region with the scale of the initially fastest growing perturbation in the plane of the maximum field $L=2\pi/\kappa=2\pi r_d[(m/3M)(E_m^4/E_p^4)]^{-1/3}$, where r_d is the Debye radius, $E_p^2=4(T_e+T_i)(m\omega^2/e^2)$ is the characteristic plasma field. Consequently, within the time $\tau=10\gamma^{-1}=5\omega_p^{-1}[(m/3M)(E_m^2/E_p^2)]^{-1/2}$ of the instability, the electromagnetic wave penetrates into the plasma to a distance on the order of the skin-layer thickness. The stratification process occurs in the succeeding skin layers and leads either to a bleaching of the plasma or to a shift of the region of reflection of the electromagnetic wave into so dense plasma, that the condition of the modulation instability of oscillations with period L ceases to hold. Of course, the described model is valid only for the initial stage of the interaction process, during which effects of the redistribution of the concentration in the entire region occupied by the field do not play any role.

We represent the initial system of equations for slow amplitudes of a field with frequency close to the plasma frequency, $\mathbf{E}=\vec{\mathcal{E}}(\mathbf{r},t)e^{i\omega t}$, with allowance for the small density perturbation produced under the influence of the electromagnetic pressure, and with allowance for the linear spatial dispersion, in the form

$$\text{rot rot}\,\vec{\mathcal{E}}=k_0^2(\epsilon_0(z)-n)\vec{\mathcal{E}}+i3r_d^2\,\text{grad div}\,\vec{\mathcal{E}}-2i\frac{\omega}{c^2}\frac{\partial}{\partial t}, \tag{1}$$

$$\frac{\partial^2 n}{\partial t^2}-v_s^2\frac{\partial^2 n}{\partial x^2}=v_s^2\frac{\partial^2}{\partial x^2}|\mathcal{E}|^2. \tag{2}$$

Here $\vec{\mathcal{E}}=\mathbf{E}/E_p$, $\mathcal{H}=\mathbf{H}/E_p$, n is the relative perturbation of the concentration, and $v_s=(T_e/M)^{1/2}$ is the velocity of the ion sound.

In this case, the electric field is in fact a superposition of the fields of the electromagnetic wave and of plasma oscillations having essentially different spatial scales. This enables us to use an averaging method for the solution of (1) and (2). We solve first the problem of the excitation of plasma oscillation in the section $z=$const by the incident-wave field $\vec{\mathcal{E}}=\mathbf{x}_0\mathcal{E}(z,t)$, $\vec{\mathcal{H}}=\mathbf{y}_0H(z,t)$ described by the x-component of Eq. (1), in which we assume that the electric induction vector $D_x=k_0^{-2}(\partial^2E_x/\partial z^2)$ is constant, and by Eq. (2), and the average Eq. (1) over the spatial period of the plasma oscillations, i.e., we find $\bar{E}$ as a function of D_x and consider the propagation of the electromagnetic wave, introducing the concept of the effective dielectric constant

$$\epsilon_{\text{eff}}=[\bar{E}(D_x)/D_x]^{-1}. \tag{3}$$

The right-hand side of (3) also depends on the value of

ϵ_{eff}, so that this relation is a transcendental equation for the determination of ϵ_{eff}. Obviously, the electromagnetic wave is a propagating one in the region $\epsilon_{eff} > 0$.

To obtain an estimate of the characteristic bleaching parameters, we consider the stationary solution of the problem, without dwelling in detail on an analysis of the processes of establishment and stability of this state.[2)] The equation for the plasma oscillations can be represented in this case in the form

$$\frac{d^2 v}{d\zeta^2} - v + v^3 = v_0 ,$$

where

$$v = |\epsilon|^{-1/2}\mathcal{E}, \quad \zeta = \sqrt{|\epsilon|}\, x/\sqrt{3}\, r_d, \quad v_0 = |\epsilon|^{-3/2} k_0^{-2} \frac{\partial^2 \mathcal{E}}{\partial z^2},$$
$$|\epsilon| = |\epsilon_0| + \overline{v^2} \qquad (4)$$

the bar denotes averaging over x. The solution of Eq. (4), in which we are interested, takes in the region $v_0 > \frac{2}{3}\sqrt{3}$ the form of a periodic sequence of narrow peaks with period $\sqrt{(|\epsilon|/3)}(L/r_d)$. As a result of the averaging of the distribution of the field $E_x(x)$ we obtain a certain equation for the effective dielectric constant, which is valid in the region of the instability. Without dwelling on the details, we present the upper bound that this equation imposes on the condition for the bleaching of a bounded plasma layer

$$\frac{E_m^2}{E_p^2} > \frac{1}{64}\left(\frac{2M}{m}\right)^{1/2} |\epsilon_M|^{3/2}, \qquad (5)$$

where ϵ_M is the value of ϵ at the maximum of the layer. The quantity E_m^2 can be connected with the amplitude E_0 of the wave incident from vacuum, by using for example the relation $E_m^2 = 3.6(k_0 l)^{1/3} E_0^2$, where l is the characteristic dimension of the inhomogeneity of the unperturbed plasma in the region of the reflection point.

Estimates of the characteristic parameters of the bleaching (the threshold amplitude of the wave and the bleaching time) agree well with the experimental data.[3,4] Moreover, the hypothesis that narrow waveguides are produced was tested by probing the perturbed region of the weak trial wave with linear polarization of the electric field $\mathbf{E}_c$—in the presence of an intense wave the plasma turned out to be transparent to a probing wave with $\mathbf{E}_c \parallel \mathbf{E}_0$ and opaque to a wave with orthogonal field polarization.[3] For a final answer to the question of the feasibility of this mechanism, however, it is necessary to experiment further on certain interaction features that follow from the proposed model.

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[1)]It can be shown that in the case of modulation instability the traveling plasma waves are unstable to "opposing" perturbations. This instability leads to formation of almost standing distributions of the plasma oscillations.

[2)]A stationary distribution of Langmuir oscillations need not necessarily be established in the case of modulation instability. For example, in the absence of effective dissipation mechanisms the level of the Langmuir oscillations fluctuates about a certain mean value; analogous oscillations but of much smaller amplitudes have been observed also in the plasma-density distribution.[6,7]

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