

Gauge fields on superspaces with different holonomy groups

V. P. Akulov, D. V. Volkov, and V. A. Soroka

Physico-technical Institute, Ukrainian Academy of Sciences

(Submitted September 1, 1975)

Pis'ma Zh. Eksp. Teor. Fiz. 22, No. 7, 396-399 (5 October 1975)

We discuss variants of a generally-covariant theory of superfields with nonzero values of the torsion tensors and the curvature tensor.

PACS numbers: 11.10.Np

1. A generally-covariant theory of superfields was recently proposed^[1,2] for a space with coordinates $z^A = (x^\mu, \phi^\alpha, \phi^{\dot{\alpha}})$, (x^μ are the usual spatial coordinates, and ϕ^α and $\phi^{\dot{\alpha}}$ are anticommuting spinor coordinates).¹⁾

The generalized Einstein equation for the superspace take the form^[2]

$$R_{AB} = 0, \quad (1)$$

where $R_{AB} = R_{AC;B}^C$ and $R_{AB;C}^D$ is the curvature tensor of the superspace.

In the expansion of the superfield of the metric tensor g_{AB} in terms of ordinary fields, Eq. (1) leads to second-order equations both for the fields with integer spins and for fields with half-integer spin. The latter circumstance is due to the absence from the structure constants of a maximal holonomy group²⁾ of Riemannian superspace satisfying Eq. (1), of quantities that can play the role of the matrices γ_μ in the equations for

fields with half-integer spin, and is a shortcoming of the theory.

We wish to call attention in this paper to the existence of generally-covariant theories free of the foregoing shortcoming.

2. The Cartan equation for a superspace with coordinates z_A can be written in the form^[5]

$$d\omega^A(\delta) + \omega^B(\delta) \wedge \Gamma_B^A(d) = \frac{1}{2} \omega^B(\delta) \wedge \omega^C(d) T_{CB}^A, \quad (2)$$

$$d\Gamma_A^B(\delta) + \Gamma_A^C(\delta) \wedge \Gamma_C^B(d) = \frac{1}{2} \omega^C(\delta) \wedge \omega^D(d) R_{DC;A}^B, \quad (3)$$

where T_{CB}^A and $R_{CD;A}^B$ are the torsion and curvature tensors. The differentiations and the products of the forms in expressions (2) and (3) are external and are determined, just for ordinary spaces, by alternating the differentials d and δ .

we choose the holonomy group of the considered superspace the Poincaré group supplemented by translation of the spinor variables. In this case only the components $\Gamma_\alpha^{\dot{B}}(d)$, $\Gamma_{\dot{A}}^B(d)$, and $\Gamma_\mu^\nu(d)$ of the differential form of connectivity differ from zero and satisfy the relations

$$2g^{\mu\rho}\Gamma_\rho^\nu(d) = \Gamma_\beta^{\dot{A}}(d)(\sigma^{\mu\nu})_{\dot{A}}^{\dot{B}} + \Gamma_{\dot{\beta}}^{\dot{A}}(d)(\sigma^{\mu\nu})_{\dot{A}}^{\dot{B}}, \quad (4)$$

where

$$\sigma^{\mu\nu} = \frac{1}{2}(\sigma^\mu\sigma^\nu - \sigma^\nu\sigma^\mu).$$

The invariant action integral for the superfields that determine the differential forms $\omega^A(d)$ and $\Gamma_B^A(d)$, can be represented in the form

$$\int L(R, T) \mathbb{W} \Pi dz^A, \quad (5)$$

where L is an invariant function of the curvature and torsion tensors, and

$$\mathbb{W} = \text{Det} |\omega_A^B|, \quad (6)$$

where the matrix ω_A^B is determined by the form coefficients²⁾ $\omega^B(d) = dz^A \omega_A^B$.

In the simplest case, L is a function of the following invariant quantities

$$R_1 = i [R_{\mu\nu;\alpha}^{\dot{B}}(\sigma^{\mu\nu})_{\dot{B}}^{\dot{A}} - R_{\mu\nu;\dot{A}}^{\dot{B}}(\sigma^{\mu\nu})_{\dot{B}}^{\dot{A}}], \quad (7a)$$

$$R_2 = i [R_{\alpha\beta;\gamma}^{\dot{B}}\epsilon^{\alpha\gamma} - R_{\dot{\alpha}\dot{\beta};\dot{\gamma}}^{\dot{B}}\epsilon^{\dot{\alpha}\dot{\gamma}}], \quad (7b)$$

$$T_1 = i T_{\alpha\dot{\beta}}^{\dot{B}}(\sigma_\mu)^{\dot{B}}_{\dot{A}}, \quad (7c)$$

$$T_2 = i [T_{\mu\dot{A}}^{\dot{B}}(\sigma^\mu)^{\dot{B}}_{\dot{A}} - T_{\mu\dot{A}}^{\dot{B}}(\sigma^\mu)^{\dot{B}}_{\dot{A}}]. \quad (7d)$$

3. We consider the case when

$$L = a_1 R_1 + a_2 R_2. \quad (8)$$

The variation of the superfields for individual terms of the action integral (5) and (8) takes the form

$$\begin{aligned} (\widetilde{R_{DC;A}^B \mathbb{W}}) = & \left\{ \left[-\omega_C^F R_{FD;A}^B + \frac{1}{2} (-)^F \widetilde{\omega}_F^F R_{CD;A}^B - (-)^F T_{CF;F} \widetilde{\Gamma}_{DB}^A \right. \right. \\ & \left. \left. + \frac{1}{2} T_{CD}^F \widetilde{\Gamma}_{FA}^B \right] - (-)^{CD} (C \leftrightarrow D) \right\} \mathbb{W}, \end{aligned} \quad (9)$$

where the tilde over a quantity denotes its variation, and the factors of the form $(-)^F$ determine the signs of the corresponding terms, depending on their graduation.

The equations for the superfields are established by substituting the expression (9) in (7a, b) and by equating to zero the coefficients of the independent variations $\widetilde{\omega}_A^B$, $\widetilde{\Gamma}_{A\alpha}^B$, and $\widetilde{\Gamma}_{A\dot{\alpha}}^B$.

An essential difference between the equations obtained from the Lagrangian (8) and the generalized Einstein equations considered in^[1,2] is the fact that as a result of the weakening of the holonomy group not all the components of the torsion tensor are equal to zero.⁴⁾ In

particular, the Lagrangian (8) contains a term corresponding to a superspace with constant torsion tensor

$$T_{\alpha\dot{\beta}}^{\dot{B}}{}^\mu = i t (\sigma^\mu)_{\alpha\dot{\beta}}^{\dot{B}} \quad (10)$$

for which

$$\Gamma_{A\alpha}^{\dot{B}} = \Gamma_{A\dot{\alpha}}^B = 0, \quad \omega^A = d\phi^A, \quad \omega^{\dot{A}} = d\phi^{\dot{A}}$$

and

$$\omega^\mu(d) = dx^\mu + \frac{t}{2i} (\phi \sigma^\mu d\phi^+ - d\phi \sigma^\mu \phi^+). \quad (11)$$

Such a superspace, first introduced in^[6,7], constitutes the basis of different variants of supersymmetry theory.

For solutions that differ little from (11), the Lagrangian leads to typically supersymmetric structure of the superfields.

4. In the presence of invariant combinations of the torsion tensor in the Lagrangian, the variations of such combinations can be expressed in terms of the quantity

$$\begin{aligned} (\widetilde{T_{BC;A}^F \mathbb{W}}) = & \left\{ \left[-\widetilde{\omega}_B^F T_{FC;A}^A - \frac{1}{2} (-)^F \widetilde{\omega}_F^F T_{BC;A}^A + (-)^{AC+F} \omega_B^A T_{FC;F}^A \right. \right. \\ & \left. \left. + \frac{1}{2} (-)^{(A+F)(B+C+F)} \widetilde{\omega}_F^A T_{BC;F}^F + \widetilde{\Gamma}_{BC;A}^A \right] - (-)^{BC} (B \leftrightarrow C) \right\} \mathbb{W} \\ \widetilde{\mathbb{W}} = & (-)^F \widetilde{\omega}_F^F \mathbb{W}. \end{aligned} \quad (12)$$

The requirement that the space with a constant vector torsion be admitted by the equations of motion leads in this case to definite relations between the constants at different degrees of the torsion tensor (7c) in the Lagrangian.

¹⁾For a detailed bibliography on supersymmetry see^[3], where no reference is made, however, to the pioneering paper^[4].

²⁾The holonomy group is a group of transformations of a reference frame in parallel transfer of the latter along an infinitesimal closed contour.

³⁾A discussion of the properties of the determinants of matrices that contain anticommuting parameters is contained in^[1].

⁴⁾We note that if we require in addition that all the components of the torsion tensor be equal to zero, then the latter holds for the Lagrangian (8) only under the condition that all the components of the curvature tensor also vanish.

¹⁾R. Arnowitt, P. Nath, and B. Zumino, Phys. Lett. 56B, 81 (1975).

²⁾R. Arnowitt and P. Nath, Phys. Lett. 56B, 117 (1975).

³⁾B. Sumino, CERN Preprint TH.1901, 1974.

⁴⁾Yu. A. Gol'fand and E. P. Likhtman, ZhETF Pis. Red. 13, 452 (1971) [JETP Lett. 13, 323 (1971)].

⁵⁾D. V. Volkov and V. A. Soroka, Teor. Mat. Fiz. 20, 291 (1974).

⁶⁾D. V. Volkov, V. P. Akulov, ZhETP Pis. Red. 16, 621 (1972) [JETP Lett. 16, 438 (1972)]; Teor. Mat. Fiz. 18, 39 (1974).

⁷⁾Abdus Salam and J. Strathdee, Nucl. Phys. B76, 477 (1974).