Spontaneous symmetry breaking. Possible values of the spin of a Goldstone particle

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The possibility of spontaneous breaking of continuous symmetry and the admissible values of the Goldstone-particle spins are considered for space-time of arbitrary dimension at zero and finite temperature.

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It is known that spontaneous breaking of continuous symmetry in local quantum field theory is closely connected with the existence of massless particles (the Goldstone theorem[1–4]). The spin (modulus of helicity) of a massless Goldstone particle is equal to zero in the case of Lie groups of internal symmetry, and is to $\frac{1}{2}$ for groups containing spinor generators (supersymmetry groups).[5,6] It is of interest to ascertain what values of the spin of a Goldstone particle are possible in general or equivalently, what types of symmetry can be spontaneously broken. It was shown in[7] that in four-dimensional space-time, in quantum theory without an indefinite metric, at zero temperature, only scalar and spinor ($S_G=\frac{1}{2}$) Goldstone particles are possible.

We consider in this paper, within the framework of quantum field theory, the possibility of spontaneous breaking of continuous symmetry, and obtain the admissible values of the Goldstone-particle spin for a space-time of arbitrary dimension at both zero and finite temperature.

We consider first the quantum field theory in n-dimensional space-time (with one variable x^0 and $n-1$ spatial coordinates $\mathbf{x}$), satisfying the Wightman axioms, at zero temperature. We assume that the generators of the group of automorphisms of the algebra of the observables can be represented in the form of integrals of the corresponding currents: $Q_{(\ldots)r}=\int dx^0 d\mathbf{x} f_r(\mathbf{x}) g(x^0) \times J_{0(\ldots)}(x)$, where $(\ldots)$ denotes the aggregate of Lorentz indices (tensor and spinor), and $f_r(\mathbf{x})$ and $g(x^0)$ are smooth functions with compact carrier, defined in analogy with the case $n=4$.[3,7]

The necessary condition for spontaneous breaking of the continuous symmetry generated by the current $J_{\mu(\ldots)}(x)$ is, as shown in[3] that the norm of the vector $E_0 Q_{(\ldots)} r|0\rangle$ become infinite as $r\to\infty$ ($|0\rangle$ is the vacuum vector, E_0 is the operator of projection on states with $p^2=0$, and p is the n-dimensional momentum). Taking into account, first, the fact that $\langle 0|J^*_{\mu(\ldots)} E_0 J_{\nu(\ldots)}|0\rangle \times (p) = p_0^{2dJ-n+2}\Pi_{\mu\nu(\ldots)}(p)\delta(p^2)$, where $\Pi_{\mu\nu(\ldots)}(p)$ is a dimensionless (matrix) function of the momentum and d_J is the scale dimensionality of a massless field having the same transformation properties relative to the Lorentz group as the current $J_{\mu(\ldots)}(x)$, and taking into account, second, the properties of the Fourier transforms of the functions $f_r(\mathbf{x})$ and $g(x^0)$, we obtain

$$\lim_{r\to\infty} \| E_0 Q_{(\ldots)r}|0>\|^2 = \lim_{r\to\infty} \int d^n p\, |\tilde{f}_r(\mathbf{p})|^2 |\tilde{g}(p_0)|^2 \times <0|J^*_{0(\ldots)} E_0 J_{0(\ldots)}|0>(p) = \lim_{r\to\infty} r^{2n-2-2d_J}\Phi, \tag{1}$$

where Φ is a finite integral.

Thus, spontaneous breaking of the symmetry generated by the current $J_{\mu(\ldots)}(x)$ is possible only under the condition

$$d_J < n-1. \tag{2}$$

Further, using the relation between the spin S_G of the Goldstone particles and the spin S_J of the current J, and recognizing that in unitary massless representations of the n-dimessional Poincaré group we have $d_J = n/2 - 1 + S_{J\,\max}$, we obtain a limitation on the possible values of the Goldstone-particle spin:

$$S_G < \frac{n}{2} - 1. \tag{3}$$

It follows from (2) and (3) that at zero temperature we get the following:

1) In two-dimensional space-time the spontaneous breaking of the continuous global symmetry is generally impossible with either tensor or spinor generators (regardless of the arguments of[8] that there exist no scalar Goldstone bosons at $n=2$).

2) In three-dimensional space-time, only scalar Goldstone particles are possible.

3) At $n=4$, scalar and spinor ($S_G=\frac{1}{2}$) Goldstone particles are possible.[7]

4) At $n>4$, Goldstone particles with high spins are possible.

We emphasize that the requirement that the metric of the state space be positive (unitarity of the representations of the Poincaré group) plays a most important role in the derivation of condition (3). If we forgo this requirement, then no limitations are imposed on the values of the Goldstone-particle spin.

We consider now the quantum field theory at finite temperature T (see, e.g.,[9,10]). At $T>0$ we get $\langle J^*_{\mu(\ldots)} E_0 J_{\nu(\ldots)}\rangle_{T>0}(p) = \langle 0|J^*_{\mu(\ldots)} E_0 J_{\nu(\ldots)}|0\rangle_{T=0}(p) + (\exp[p_0/T]\mp 1)^{-1} A_{\mu\nu(\ldots)}(p)\delta(p^2)$, where $A_{\mu\nu(\ldots)}(p)\delta(p^2)$

has the same degree of momentum homogeneity as $\langle 0 | J^*_{\mu(\ldots)} E_0 J_{\nu(\ldots)} | 0 \rangle (p)$. The behavior of $\langle J^*_{\mu(\ldots)} \times E_0 J_{\nu(\ldots)} \rangle_{T>0}(p/r)$ as $p \to \infty$ is different for spinor and tensor currents. For spinor currents (half-integer S_J) both terms in $\langle J^*_{\mu(\ldots)} E_0 J_{\nu(\ldots)} \rangle_{T>0}(p/r)$ have the same degree of singularity as $r \to \infty$, whereas for tensor currents the second term is more singular and therefore $\langle J^*_{\mu(\ldots)} E_0 J_{\nu(\ldots)} \rangle_{T>0}(p/r)$ behaves like $r^{-(2dJ-n-1)} |\mathbf{p}|^{-1} A_{\mu\nu(\ldots)}(p)\delta(p^2)$ as $r \to \infty$.

As a result, for spinor Goldstone particles the conditions (2) and (3) remain in force also at $T > 0$, whereas for Goldstone bosons at $T > 0$ we have

$$S_{\text{G.b.}}(T > 0) < \frac{n}{2} - \frac{1}{2} \quad . \tag{4}$$

Thus, in 4-dimensional space-time at $T > 0$, in addition to scalar and spinor ($T = 0$) particles it is possible also to have Goldstone particles with unity spin. In two- and three-dimensional space-time, spontaneous breaking of continuous symmetry at $T > 0$ is impossible, since the scalar Goldstone particles allowed by condition (4) are forbidden at $n = 2$ or 3 by the requirement that the field fluctuations be finite.[8,11]

We note in conclusion that we have considered the case of a zero chemical potential μ. If $\mu \neq 0$ and $T > 0$, it is easy to verify that conditions (2) and (3) are valid.

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