Nonlinear damping of helicons in metals

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In pure metals with unequal electron and hole densities, low-frequency electromagnetic excitations (helicons) can propagate at low temperatures and in the presence of a magnetic field.[1-3] If $kR < 1$ (k is the wave vector and R is the Larmor radius), and the helicon propagates along a magnetic field $\mathbf{H}_0 \parallel oz$ directed along a symmetry axis of the crystal, then the damping is due only to collision processes. In the case of propagation at an angle to the magnetic-field direction in the region $kR < 1$, $|k_z| l \gg 1$ (l is the mean free path), the main mechanism is collisionless magnetic Landau damping, the linear theory of which was developed by Kaner and Skobov in[4]. In this communication we present the main results of the nonlinear theory of magnetic Landau damping, a detailed exposition of which will be presented in a separate paper.

Let us dwell first on the physical aspect of the phenomenon. Let the helicon propagate at an angle θ to the magnetic field in a metal with an isotropic quadratic dispersion law, and let the wave vector $\mathbf{k}$ lie in the yz plane. The electric field of the wave lies in the xy plane and its components are connected by the relation $E_x = -iE_y \cos\theta$. The magnetic field $\mathbf{H}$ is circularly polarized in a plane perpendicular to $\mathbf{k}$. We assume that $kR \ll 1$ and $\omega \ll \omega_c$, and then the motion of the electron constitutes rapid rotation on a Larmor orbit whose center moves in fields that vary slowly in space and in time. From the equation of motion of an individual particle in a field it is seen that in the region $kR \ll 1$ the helicon interacts effectively with particles whose orbit centers move in the constant-phase plane of the wave and for which the condition $k_z v_z = \omega_0$ is satisfied. Under the action of the electric-field component E_x and the magnetic-field component H_y, the center of the orbit is accelerated in the direction of z and the particle energy changes. The magnetic force lines of the field $\mathbf{H}_0 + \mathbf{H}$ form a system of twisted moving magnetic bottles so arranged that the regions of condensation and rarefaction of the force lines alternate in planes perpendicular to $\mathbf{k}$. The magnetic bottles trap the resonant electrons. The centers of the Larmor orbits of the trapped particles oscillate in the bottles with characteristic frequencies $\omega_0 = k_z v_F \sqrt{(h \sin\theta)/2}$, where v_F is the Fermi velocity, $h = H/H_0$ and H is the amplitude of the wave. If the oscillation frequency in the bottle is much less than the collision frequency τ^{-1} then the trapping is negligible and the linear theory is valid. On the other hand if the particle does not have time to be scattered during the period of the oscillation in the bottle, then the trapping is effective. In this case, owing to modulation of the velocity v_z, the conditions for resonant interaction of the particles with the wave are violated and the absorption coefficient decreases.

Since the spectrum of the helicon in the region $kR \ll 1$ is formed by all the electrons of the Fermi surface, and the trapped electrons lie in a narrow region $\bar{v}_z \sim \omega_0/k_z \ll v_F$, the real part of the helicon spectrum is not changed by the trapping

The damping coefficient can be calculated on the basis of the kinetic equation, which takes the following form in a coordinate system moving along the z axis with velocity $v_0 = v_{ph}/\cos\theta$ (v_{ph} is the phase velocity of the wave):

$$v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} + \frac{e}{c}[\mathbf{v} \times (\mathbf{H}_0 + \mathbf{H})] \frac{\partial f}{\partial \mathbf{p}} + \hat{I}\{f\} = 0. \quad (1)$$

In (1) we have neglected the derivative $\partial f/\partial t$, which is proportional to the small damping coefficient of the wave. In our coordinate system, the magnetic field of the wave coincides with the field in the laboratory frame, accurate to terms of order $(v_{ph}/c)^2$.

$+g_p(y,z)$, where G_0 is the equilibrium distribution function. Changing over to the variables $\xi = k_z z + k_y y$, v_z, $\epsilon = m(v_z^2 + v_\perp^2)/2$ and ϕ, where ϕ is the angle between $\mathbf{v}_\perp$ and the x axis, we get

$$(k_z v_z + k_y v_\perp(\epsilon, v_z)\sin\phi)\frac{\partial g}{\partial \xi} - \omega_c h v_\perp(\epsilon, v_z)[\cos\phi\cos\xi\cos\theta$$

$$-\sin\phi\sin\xi]\frac{\partial g}{\partial v_z} + \omega_c\frac{\partial g}{\partial\phi} + \hat{I}\{g\} = -F_0'(v + v_0)\frac{v_{ph}eH}{c}$$

$$\times\left[v_\perp(\epsilon, v_z)\cos\phi\cos\xi - \frac{v_\perp(\epsilon, v_z)}{\cos\theta}\sin\phi\sin\xi\right]. \quad (2)$$

Here $v_\perp(\epsilon, v_z) = \sqrt{2m\epsilon - v_z^2}$. We have left out from the coefficient preceding the derivative $\partial g/\partial\phi$ the terms of order $h \ll 1$, which are of no importance in what follows.

Equation (2) is solved by the method of characteristics. To find the characteristics we used the method of averaging over the fast rotations on the Larmor orbit.[5] Along the trajectory we have $\xi = \bar{\xi} - k_y R\cos\phi$, where $\bar{\xi}$ describes the slow motion of the center of the orbit. Substituting this expression in the right-hand side of (2) and expanding in terms of $k_y R \ll 1$ up to terms of first order inclusive, we obtain an expression in which it is necessary to separate the term describing the magnetic Landau damping; this term is equal to $-F_0'(v_{ph}eH/2c)v_\perp(\epsilon, v_z)k_y R\sin\bar{\xi}$. To determine that part of the function g which describes the magnetic Landau damping (we designate it by g_1), we need retain only this term in the right-hand side, by virtue of the linearity of (2). We note that the particles that interact effectively with the wave lie in a narrow velocity region $|v_z| \ll v_F$, and therefore we can confine ourselves in the collision integral only to the outward drift processes, and express the integral in the form $\hat{I}\{g_1\} = (1/\tau)g_1$.

To solve the equation for g_1, the particles are naturally subdivided into two groups, trapped and untrapped. The former execute finite motion in $\bar{\xi}$, and the latter infinite motion. The distribution functions of the trapped and untrapped particles can be represented in the form of infinite series containing elliptic integrals of the first kind. They are quite complicated in form and are therefore not presented here.

The helicon damping coefficient is calculated from the formula for the change of the wave energy averaged over the period, $\partial\tilde{W}/\partial t = -2\,\mathrm{Im}\,\tilde{W} = \tilde{\mathbf{j}}\times\tilde{\mathbf{E}}$. The superior wavy line means averaging over the period, and the average energy is equal to

$$\tilde{W} = \frac{1}{16\pi}\left\{\frac{d}{d\omega}(\omega\epsilon_{ik})E_i^* E_k + |\mathbf{H}|^2\right\}.$$

absorption coefficient Γ_{nl} to the linear one Γ_{lin} is equal to $\Gamma_{nl}/\Gamma_{lin} = \gamma_t + \gamma_{ut}$, where γ_t and γ_{ut} are respectively the contributions of the trapped and untrapped particles, equal to

$$\gamma_t = 128\pi^2\int_1^{+\infty}\frac{d\kappa}{\left(\kappa K\left(\frac{1}{\kappa}\right)\right)^3}\sum_{n=1}^{\infty}\frac{\left(n-\frac{1}{2}\right)^2 q^{2n-1}\left(\frac{1}{\kappa}\right)}{\left(1+q^{2n-1}\left(\frac{1}{\kappa}\right)\right)^2}\frac{\tau^{-1}\omega_0}{\tau^{-2}+\left(\frac{\pi\omega_0\left(n-\frac{1}{2}\right)}{K\left(\frac{1}{\kappa}\right)}\right)^2} \quad (3)$$

$$\gamma_{ut} = 128\pi^2\int_0^1\frac{d\kappa}{\kappa^6 K^2(\kappa)}\sum_{n=1}^{\infty}\frac{n^2 q^{2n}(\kappa)}{(1+q^{2n}(\kappa))^2}\frac{\tau^{-1}\omega_0}{\tau^{-2}+\left(\frac{\pi n\omega_0}{\kappa K(\kappa)}\right)^2}. \quad (4)$$

Here $K(\kappa)$ is a complete elliptic integral of the first kind, and $q(\kappa) = \exp\{-\pi K(\sqrt{1-\kappa^2})/K(\kappa)\}$. If $\omega_0\tau \ll 1$, then it follows from (3) and (4) that $\gamma_{ut} = 1$ and $\gamma_t \sim \omega_0\tau \ll 1$. This means that the trapping is insignificant and the linear theory is valid. If $\omega_0\tau \gg 1$, then $\gamma_y + \gamma_{ut} \approx A/\omega_0\tau$, where A is a coefficient on the order of unity. Numerical calculation yields the value $A \approx 2$. It is of interest to note that an analogous result is obtained for the ratio of the nonlinear absorption coefficient of longitudinal sound to the linear one in conductors and superconductors.[6,7]

We present estimates of the magnitude of the effect. Assuming $\tau = 5\times10^{-10}$ sec, $H_0 = 4\times10^4$ Oe, and $kR = 0.3$, and also that v_F and the carrier density have values typical of metals, we find that at a power flux density 1 W/cm^2 delivered to the sample the parameter $\omega_0\tau$ is equal to 3, and the absorption coefficient decreases by approximately one-half. It can be shown that the ensuing heating is negligible.

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