SU(4) scheme of weak interactions and leptonic decays of baryons

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We consider the $SU(4)$ scheme of weak interactions and investigate on its basis the possible leptonic decays of the baryons 20 $(1/2^+)$.

One of the realistic theoretical interpretations of the recently discovered narrow resonances $\psi(3.1)$ and $\psi(3.7)$ is to introduce a charmed quark p' into the systematics of the elementary particles. As a result of the analysis of the theoretical schemes with a charmed quark we arrive at a large number of new interesting facts (see[1]). Thus, it seems quite natural to broaden the symmetry of the strong interactions to the framework of broken $SU(4)$. Accordingly, in connection with the appearance of new weak currents $p' \rightarrow n$ and $p' \rightarrow \lambda$, it is necessary to review also the scheme of weak-interaction physics (see, e.g.,[2]). It is obvious that the most natural generalization of the Cabibbo theory is an attempt to formulate weak interactions in the language of the $SU(4)$ group, we shall thus propose, for example, that the vector and axial currents jointly with the electromagnetic current belong to a regular representation. We note that the unified (GIM) model of weak and electromagnetic interactions,[3] constructed on the principle of local gauge invariance with spontaneous breaking, can be regarded within the framework of this hypothesis, subject to one correction, namely that the vector interactions belong to the mixed representation $15+1$.

Thus, consideration of the charmed quark leads to the a complicated experimental situation: it is necessary to search for new particles, both mesons and baryons, and to study the nature of their strong, weak, and electromagnetic interactions. We note that obser-

vation of these particles is made very difficult by the increased number of hadronic decay channels, and obviously calls for special new theoretical and experimental methods. In all probability one of the most successful methods will be a study of their interaction with leptons, and particularly the singularities of their production in interactions between high-energy neutrino beams with matter.

In this article we shall discuss the interaction between the particles connected with the new quantum number "charm" and leptons, within the framework of $SU(4)$ symmetry; this turns out to be most important both for the analysis of leptonic decays of charmed particles and for the study of their production in lepton beams. More specifically, we consider transitions of baryonic states from the 20-plet $(\frac{1}{2}^+)$ to states of the same 20-plet via a charged current, which according to our hypothesis belongs to the regular $SU(4)$ representation and is given by

$$J_\mu' = aJ_\mu^{1+i2} + bJ_\mu^{4+i5} + cJ_\mu^{11+i12} + dJ_\mu^{13+i14}. \tag{1}$$

To calculate the Clebsch-Gordan coefficients corresponding to the matrix elements $\langle \overline{20}|J_\mu|20\rangle$ we use the method of generating invariants,[4] which makes it also possible to ascertain the selection rules. The method of generating invariants $J(20, 15, \overline{20})$ made up of the components of the bases of the irreducible representations $D(20)$, $D(15)$ and $D(\overline{20})$. Since the product of the

sible to construct only two independent generating invariants, corresponding in the analogous situation for the $SU(3)$ group to the presence of F and D bonds. The table lists basis of the state vectors of the 20-plet, represented in the form of homogenous polynomials.[4] To determine the basis, we used infinitesimal operators realized formally as differential operators in the polynomial space.

$C = 0$

$p = x_1 v_{12}$

$n = x_2 v_{12}$

$\Sigma^+ = x_1 v_{13}$

$\Sigma^\circ = \frac{1}{\sqrt{2}}(x_1 v_{23} - x_2 v_{31})$

$\Sigma^- = x_2 v_{23}$

$\Lambda^\circ = \frac{1}{\sqrt{6}}(x_1 v_{23} + x_2 v_{31} - 2 x_3 v_{12})$

$\Xi^\circ = x_3 v_{13}$

$\Xi^- = x_3 v_{23}$

$C = 1$

$A^+ = \frac{1}{\sqrt{2}}(x_3 v_{14} + x_1 v_{43})$

$A^\circ = \frac{1}{\sqrt{2}}(x_3 v_{24} + x_2 v_{43})$

$C_\circ^+ = \frac{1}{\sqrt{2}}(x_1 v_{24} - x_2 v_{41})$

$C = 1$

$C_1^{++} = x_1 v_{14}$

$C_1^+ = \frac{1}{\sqrt{2}}(x_1 v_{24}) - x_2 v_{41})$

$C_1^{\circ} = x_2 v_{24}$

$S^+ = \frac{1}{\sqrt{2}}(x_3 v_{14} - x_1 v_{43})$

$S^\circ = \frac{1}{\sqrt{2}}(x_3 v_{24} - x_2 v_{43})$

$T^\circ = x_3 v_{34}$

$C = 2$

$X_u^{++} = x_4 v_{14}$

$X_d^+ = x_4 v_{24}$

$X_s^+ = x_4 v_{34}$

We note that the forms $x_i v_{jk}$ ($v_{jk} = x_j y_k - x_k y_j$; $i, j, k = 1, 2, 3, 4$) make up the so-called "symmetrical" basis of the representation 20. The basis of the contragradient representation $\overline{20}$ is constructed in the usual manner from the contravariant quantities $z^{ic_{jk}}$; for the 15-plet, the basis was chosen in the form $x_i \xi^j$ (with the condition $x_i \xi^i = 0$). The coefficients of the expansion of the generating invariants $J_{1,2}(20, 15, \overline{20})$ in terms of the basis functions are, by definitions, Clebsch-Gordan coefficients.

As a result of straightforward but cumbersome calculations, in accordance with the definitions of the particles given in the table, we write the vector part of the matrix element for the 20-plet of baryons in the form

$$a\, F_v(n\tilde{p} + \sqrt{2}\,\Sigma^\circ \tilde{\Sigma}^+ + \sqrt{2}\,\Sigma^- \tilde{\Sigma}^\circ + \Xi^\circ \tilde{\Xi}^\circ + 3A^\circ \tilde{A}^+ + \sqrt{2}\,C_1^\circ \tilde{C}^{++} + \sqrt{2}\,C_1^\circ \tilde{C}_1^+$$

$$+ S^\circ \tilde{S}^+) + D_v(n\tilde{p} + \frac{2}{\sqrt{6}}\,\Sigma^- \tilde{\Lambda}^\circ - \frac{2}{\sqrt{6}}\,\Lambda^\circ \tilde{\Sigma}^+ - \Xi^- \tilde{\Xi}^\circ + C_1^\circ \tilde{C}_\circ^+ - S^\circ \tilde{A}^+ - 2A^\circ \tilde{A}^+$$

$$- \sqrt{2}\,C_\circ^+ \tilde{C}_1^{++} - A^\circ \tilde{S}^+ - X_d^+ \tilde{X}_u^{++}) \Big]_{\Delta s = 0}^{\Delta c = 0} + b\Big[F_v - \frac{\Sigma^\circ \tilde{p}}{\sqrt{2}} - \frac{3}{\sqrt{6}}\,\Lambda^\circ \tilde{p} - \Sigma^- \tilde{n}$$

$$+ \frac{3}{\sqrt{6}}\,\Xi^- \tilde{\Lambda}^\circ + \Xi^\circ \tilde{\Sigma}^+ + \frac{\Xi^- \tilde{\Sigma}^\circ}{\sqrt{2}} + 3A^\circ \tilde{C}_\circ^+ + \sqrt{2}\,S^+ \tilde{C}_1^{++} + S^\circ C_1^+ + \sqrt{2}\,T^\circ S_u^+$$

$$+ X_s^+ X_u^{++} + D_v\Big(\frac{\Sigma^\circ \tilde{p}}{\sqrt{2}} - \frac{\Lambda^\circ \tilde{p}}{\sqrt{6}} + \Sigma^- \tilde{n} + \Xi^\circ \tilde{\Sigma}^+ + \frac{\Xi^- \tilde{\Sigma}^\circ}{\sqrt{2}} - \frac{\Xi^- \tilde{\Lambda}^\circ}{\sqrt{6}} + S^\circ \tilde{C}_\circ^+ - 2A^\circ \tilde{C}_\circ^+$$

$$- \sqrt{2}\,T^\circ \tilde{A}^+ + \sqrt{2}\,A^\circ \tilde{C}_1^{++} + A^\circ \tilde{C}_1^+ - X_s^+ \tilde{X}_u^+)\Big]_{\Delta c \neq 0}^{\Delta c = 0} + e\Big[F_v\Big(p\tilde{C}_1^{++} + \frac{n\tilde{C}_1^+}{\sqrt{2}}$$

$$- \frac{3}{\sqrt{2}}\,n\tilde{C}_\circ^+ - \sqrt{\frac{3}{2}}\,\Lambda_\circ \tilde{S}^+ - \sqrt{\frac{3}{2}}\,\Lambda_\circ \tilde{A}^+ + \frac{\Sigma^\circ \tilde{S}^+}{\sqrt{2}} - \frac{\Sigma^\circ \tilde{S}^+}{2} + \frac{3}{2}\,\Sigma^\circ \tilde{A}^+ - \frac{\Sigma^- \tilde{S}_\circ}{\sqrt{2}}$$

$$+ \frac{3}{\sqrt{2}}\,\Sigma^- \tilde{A}^\circ - \Xi^- \tilde{T}^\circ - \frac{3}{\sqrt{2}}\,A^\circ \tilde{X}_s^+ + \frac{C_1^\circ \tilde{X}_u^{++}}{\sqrt{2}} + C_1^\circ \tilde{X}_d^+ + \frac{S^\circ \tilde{X}_s^+}{\sqrt{2}}\Big) + D_v\Big(- p\tilde{C}_1^{++}$$

$$+ \frac{\Sigma^\circ \tilde{S}^+}{2} - \frac{\sqrt{3}}{2}\,\Lambda^\circ \tilde{S}^+ + \frac{\Sigma^\circ \tilde{A}^+}{2} - \frac{\Lambda^\circ \tilde{A}^+}{2\sqrt{3}} - \frac{C_1^\circ \tilde{X}_u^{++}}{\sqrt{2}} + \frac{C_\circ^+ \tilde{X}_u^{++}}{\sqrt{2}} - \frac{n\tilde{C}_1^+}{\sqrt{2}} - \frac{n\tilde{C}_\circ^+}{\sqrt{2}}$$

$$+ \frac{\Sigma^- \tilde{S}^\circ}{\sqrt{2}} - \frac{\Sigma^- \tilde{A}^\circ}{\sqrt{2}} - C_1^\circ \tilde{X}_d^+ + \Xi^- \tilde{T}^\circ - \frac{S^\circ \tilde{X}_s^+}{\sqrt{2}} - \frac{A^\circ \tilde{X}_s^+}{\sqrt{2}}\Big)\Big]_{\Delta s = 0}^{\Delta c \neq 0}$$

$$+ d\Big[F_v\Big(\Sigma^\circ \tilde{C}_1^{++} - \Sigma^\circ \tilde{C}_1^+ + \sqrt{3}\Lambda^\circ \tilde{C}_\circ^+ - \frac{\Xi^\circ \tilde{S}^+}{\sqrt{2}} - \frac{3}{\sqrt{2}}\,\Xi^\circ \tilde{A}^+ + \frac{3}{\sqrt{2}}\,A^\circ \tilde{X}_u^+$$

$$+ \Sigma^- \tilde{C}_1^\circ + \frac{\Xi^- \tilde{S}^\circ}{\sqrt{2}} - \frac{3}{\sqrt{2}}\,\Xi^- \tilde{A}^\circ + \frac{3}{\sqrt{2}}\,A^\circ \tilde{X}_d^+ + \frac{S^+ \tilde{X}_u^{++}}{\sqrt{2}} + \frac{S^\circ \tilde{X}_d^+}{\sqrt{2}} + T^\circ \tilde{X}_s^+\Big)$$

$$+ D_v\Big(- \Sigma^+ \tilde{C}_1^{++} - \Sigma^\circ \tilde{C}_1^+ + \frac{\Lambda_\circ \tilde{C}_\circ^+}{\sqrt{2}} + \frac{S^+ \tilde{X}_u^{++}}{\sqrt{2}} - \frac{A^+ \tilde{X}_u^{++}}{\sqrt{2}} - \Sigma^- \tilde{C}_1^+ + \frac{S_\circ \tilde{X}_d^+}{\sqrt{2}}$$

$$- \frac{A^\circ \tilde{X}_d^+}{\sqrt{2}} - \frac{\Xi^- \tilde{S}^+}{\sqrt{2}} + \frac{\Xi^\circ \tilde{A}^+}{\sqrt{2}} - \frac{\Xi^- \tilde{S}^\circ}{\sqrt{2}} + \frac{\Xi^- \tilde{A}^\circ}{\sqrt{2}} + T^\circ \tilde{X}_s^+\Big)\Big]_{\Delta s \neq 0}^{\Delta c \neq 0}. \quad (2)$$

The axial part is written analogously, except that F_v and D_v are replaced by new coupling constants F_A and D_A. The fact that the same coefficients F and D are present in all the terms of (2) is the consequence of hypothesis (1). Just as in the case of $SU(3)$ symmetry, we have $F_v = 1$ and $D_v = 0$, while for the axial constants we can use the values[5] $F_A \approx 0.41$ and $D_A \approx 0.65$. We note that at zero momentum transfer the effect of $SU(4)$-symmetry breaking should alter these results insignificantly (the analog of the Ademolo-Gatto theorem). Following the GIM model, we can assume $a = d = \cos\theta_c$ and $b = -c = \sin\theta_c$. On the basis of formula (2) we can obtain a sufficiently large class of relations between the amplitudes of the baryon-baryon transitions, and these can be subsequently used both to estimate the leptonic decay modes and to calculate the additional contribution made by the following processes of charmed-baryon production to the total νN-interaction cross section: $\nu + p \to C_1^{++} + \mu^-$, $\nu + n \to C_1^+ + \mu^-$, $\nu + n \to C_0^+ + \mu$. It thus follows from (2) that $a(p \to C_1^{++}) = \sqrt{2}\,a(n \to C_1^+) \sim (D - F)\sin\theta_c$ and $a(n \to C_0^+) \sim [(3F + D)/\sqrt{2}]\sin\theta_c$. The newly produced baryons will decay in the main via hadronic channels, and it is undoubtedly of interest to estimate the relative contribution of the leptonic decay. It follows from[3] that these baryons of an appreciable amplitude of decay into strange particles; thus C_1^{++}, C_1^+, and C_1^0 go over into Σ^+, Σ^0, and Σ^- respectively, and $C_0^+ \to \Lambda^0$, with $a(C_1^{++} \to \Sigma^+) = a(C_1^+ \to \Sigma^0) = a(C_1^0 \Sigma^-) \sim \cos\theta_c(F - D)$ and $a(C_0^+ \to \Lambda^0) \sim \cos\theta_c(D/\sqrt{3} + \sqrt{3}F)$.

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Erratum: *SU* (4) scheme of weak interactions and leptonic decays of baryons [JETP Lett. 22, No. 9, 229–231 (5 November 1975)]

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On page 230, left-hand column, fifth line from the bottom, the term $\Xi^0\Xi^0$ should be replaced by $\Xi^-\Xi^0$ and the term $\sqrt{2}C_1^+\widetilde{C}^{++}$ by $\sqrt{2}C_1^+C_1^{+}$. In the third line from the bottom X_a^+ should be replaced by $\widetilde{X}_d^+$. In the right-hand column of p. 230, line 11 from the top, $F=1$ and $D=0$ should be replaced by $F_\nu(0)=1$ and $D_\nu(0)=0$, respectively, and in the line 13 from the top the word "zero" should be replaced by "nonzero."