The longitudinal dimensions of a fast hadron

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Arguments are presented to show that the longitudinal dimensions of a fast hadron increase with its energy E like Em^{-2}. These dimensions determine the length of the region in which the slow partons are concentrated. A number of consequences of such a picture are discussed.

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In the parton model it is assumed that the main components of the wave function of a fast hadron of energy E are multiperipheral states with a parton spectrum $d\epsilon/\epsilon$ and with bounded transverse momenta. On the other hand, the main contributions to the inelastic processes come from interactions between partons with $\epsilon \sim m$ and the target, which can be, in particular, the slow partons of the oppositely-directed moving hadron.[1,2] The spatial picture is the following: the average transverse distances at which are located the partons with energy ϵ are of the order $[\ln(E/\epsilon)]^{1/2}$, and the longitudinal distances are assumed to be $\sim 1/\epsilon$; the hadron has the shape of a disk of radius $[\ln(E/\epsilon)]^{1/2}$ and thickness $\sim 1/m$, in which the amplitude of observing the partons with $\epsilon \sim m$ is large. Into this disk are "imbedded" disks of smaller radius and thickness, containing ever faster partons.

However, a longitudinal structure of the $1/\epsilon$ type seems quite arbitrary. Because of the customary assumption of the disk picture of the hadron, a disparity arises between the consequences of the parton picture and the reggeon-diagram scheme in those cases when the coherent properties of the wave function of the fast hadron are important. For example, the long-range correlators in impact-parameter space are $\ln(E/m)$ times larger in the case of the reggeon diagrams than in the parton model. Therefore a regime of growing cross sections[3,4] is impossible in the parton model (with lengths $\sim 1/\epsilon$). Further, it is impossible to explain the equality of the asymptotic cross sections[5] in the lab system of the composite particles, since there is no analog of the coalescence of parton chains in this system.

The question of the longitudinal dimensions of the region in which the partons are concentrated can be solved by modifying the following standard reasoning: if the states of a particle with definite momenta are used to construct a packet of length z, then those z_0 for which the state ceases to be single-particle after the packet spreads out determine the order of magnitude of the particle dimension. It is obvious that such a compression into a packet perturbs the mass of the particle by an amount $\delta M^2(z)$, and then the condition $\delta M^2(z_0) \sim M^2$ determines z_0.

If the interaction is turned off, then a parton of energy ϵ makes a contribution $\sim m^2(E/\epsilon)w_\epsilon$ to the square of the hadron mass (w_ϵ is the weight with which the parton is present in the hadron state; we assume henceforth $w_\epsilon \sim 1$). The interaction that causes the hadron to have a finite mass M^2 (i.e., that makes it stable and keeps it from breaking up into partons) compensates for this contribution to M^2. However, if the parton "ϵ" is acted upon from the outside and is compressed into a packet of length z_ϵ, then $\epsilon \to \epsilon + \delta_\epsilon$, where $\delta_\epsilon \sim 1/z \ll \epsilon$, and an increment $\approx \delta_\epsilon$ appears in M^2 and is generally speaking not offset by the interaction. Expanding the perturbed contribution from "ϵ" to M^2, which amounts to $\sim m^2E/(\epsilon + \delta_\epsilon)$, in powers of δ_ϵ, we obtain[1)]

$$\delta M_\epsilon^2 \sim \left(\frac{m^2 E}{\epsilon^2}\right)_{\sigma_\epsilon} \sim \frac{m^2 E}{\epsilon^2}\frac{1}{z_\epsilon}\,.$$

The condition $\delta M_\epsilon^2 \ll M^2$ determines the dimensions of the packets that spread out only into one-hadron states; we find that $z_\epsilon \gg E/\epsilon^2$.

From the condition $\delta M_\epsilon^2 \ll M^2$ we obtain the longitudinal dimensions of the region in which the partons of energy ϵ are concentrated, namely $L_\epsilon \sim E/\epsilon^2$. A fast hadron is therefore not a disk but a long tube of length $L \sim Em^{-2}$ and of cross section $\ln(E/m)$.[2)] This result does not change most predictions of the parton scheme, with the exception of a number of coherent effects, which we now proceed to discuss.

We consider the interaction between a hadron and a single target (assuming that the number of slow partons is $\nu \sim 1$) in two different representations: (I) As a "soft" interaction over a length Em^{-2}: within a time $\sim Em^{-2}$ there are produced $\sim \ln E$ slow partons that merge with the faster ones to form a multiperipheral chain of real particles. (II) As a superposition of "hard" interactions. The interaction with the target leads to localization (accurate to $\sim 1/m$) of the slow parton in a definite part of the tube (a superposition over such localizations yields I). This localization annihilates within a time $1/m$ the amplitude of the slow parton in the entire tube. The tube becomes unstable because the assembly and disassembly "partner" has vanished for the faster partons over the entire length of the tube. The faster partons therefore begin to decay, and after a time $\sim Em^{-2}$ we have, in particular, $\sim \ln E$ slow partons. A superposition of the thus-produced states yields real hadrons. For a single target, the two methods I and II are equivalent, since only the value of ν is of importance for σ_{tot}.

We now consider an elongated target. Assume that it consists of two hadrons a_1 and a_2 separated along z by a distance $R \gg m^{-1}$, and let the impact distances be close. At $E > Rm^2$ there can occur a coherent interaction with the entire target in the dominant inelastic processes. This is permissible within the framework of picture I, since $L \sim Em^{-2} > R$, but in picture II the time variation describes the mechanism more clearly. After the interaction with a_1, the incident hadron begins to break up into partons. As a result, the total flux of slow partons incident on a_2 increases, since new slow partons appear also in those regions of the "long hadron," which have not yet reached a_2. At $E \gg Rm^2$, the average number of partons that travel past a_2 is of the order of $\bar{n} \sim \ln(E/m)$. Since all are formed inside a Regge tube with cross section $\langle x_\perp \rangle^2 \sim (1/m^2)\ln(E/m)$, the time integral of their flux in the vicinity of a_2 is $\sim n/\langle x_\perp \rangle^2 \sim m^2$, meaning that the probability of interaction with a_2 is $w \sim \sigma_0 m^2 \sim 1$. We note that it is immaterial with which of the particles (the "close" one or the "far" one) the first interaction takes place—the picture is symmetrical. Therefore at $E \gg Rm^2$ an interaction with all the components of the elongated target takes place in the dominating processes,[3)] with a probability ~ 1, this being the analog of the coalescence of the partons in the reciprocal Lorentz system.[5]

We consider now the interaction of two fast hadrons in the c.m.s. Two tubes of length Em^{-2} collide; the value of σ_{tot} is determined only by the number of slow partons, and $\sigma_{\text{tot}} \sim 1/m^2$ at $\nu \sim 1$. We start with method II. After the first interaction, parton disintegration begins in each tube, followed by collisions of the newly produced slow partons from the different tubes. In order of magnitude, each produced slow parton (in the c.m.s.) interacts. Yet the total produced number at $\nu \sim 1$ is $\sim \ln(E/m)$. This means that the inclusive cross sections in the central region should increase by $\ln(E/m)$ times. But the rapidity density of real hadrons in the c.m.s. cannot reach $\sim \ln(E/m)$, since it is limited by the value of the total cross section. Other multiparticle correlators, however, do increase generally speaking by $\ln(E/m)$ times if their values are not bounded by additional conditions. It is precisely this phenomenon which is observed in reggeon diagrams with nonvanishing three-pomeron vertices.

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[1)]The same result is obtained from a more accurate analysis of the Fock equations for the parton wave function.

[2)]This is true only for asymptotically constant cross sections. If $\alpha(0) < 1$, then $L \sim E/\epsilon_{\min}^2 = E^{1-2(1-\alpha(0))}$. On the other hand if $\epsilon \sim E$ for all partons as $E \to \infty$, then $L \sim E^{-1}$. However, the longitudinal distances that are significant in the interaction[6] are still $\sim Em^{-2}$ if the parton transverse momenta are bound. We note that $L \sim Em^{-2}$ is the dimension of the fast hadron at a given instant. At first glance the picture contradicts the Lorentz contraction, to $1/E$, of an object whose rest dimension is $1/m$. In fact, however, the slow and fast hadrons are different objects in the sense that the Lorentz transformation of one into the other includes degrees of freedom that pertain to vacuum. In this case, the bulk of the energy, $\approx E$, is concentrated in a Lorentz-contracted region of length $\sim 1/E$; this energy, however, is not effective in the interaction.

[3)]It is stated sometimes that after the first interactions the hadron cross sections can be smaller than normal during a certain time. The arguments presented here show that in the parton scheme (meaning also in the Regge scheme) the situation is just the reverse.

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