Josephson effect in the presence of a weak electromagnetic field

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It is shown that in an S-I-S' tunnel junction, by virtue of the difference between the effective masses of the electrons in superconductors, an alternating electric field produces Joesphson-current oscillations that are not connected with the voltage drop across the junction.

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It is known under the condition $\omega < 2\Delta$ (where ω is the field frequency, Δ is the energy gap in the electron spectrum, and $\hbar = 1$) the action of a weak alternating electromagnetic field on thin superconducting films, reduces mainly to acceleration of Cooper pairs of the electronic component of the field $\mathbf{E}(t) = \mathbf{E}_0 \cos\omega t$, which is parallel to the plane of the film.[1] If the film thickness d is in this case much less than the depth of penetration λ of the electric field, and the amplitude E_0 of the latter satisfies the inequality $eE_0\xi \ll \omega$ (where ξ is the coherence length), so that the alternating current is small in comparison with the pair-breaking current,[1,2] then the behavior of the superconducting electrons in the film can be described with the aid of the following Green's functions:

$$G(\mathbf{p}; t, t') = \exp\left\{\frac{ie}{cm^*}\int_{t'}^{t}\left[\mathbf{p}\mathbf{A}(t_1) - \frac{e}{2c}A^2(t_1)\right]dt_1\right\}G_0(\mathbf{p}, t-t'); \quad (1)$$

$$F^+(\mathbf{p}; t, t') = \exp\left\{\frac{ie^2}{2m^*c^2}\left[\int_{-\infty}^{t}A^2(t_1)dt_1 + \int_{-\infty}^{t'}A^2(t_1)dt_1\right] - \frac{ie}{cm^*}\int_{t'}^{t}\mathbf{p}\mathbf{A}(t_1)dt_1\right\}$$
$$\times F_0^+(\mathbf{p}, t-t'), \quad (2)$$

where $\mathbf{A}(t) = -(c/\omega)\mathbf{E}_0 \sin\omega t$ is the vector-potential of the homogenous electric field, m^* is the effective mass of the electron, and G_0 and F_0^+ are the normal and anomalous Green's functions in the absence of a field.

We consider the Josephson effect in a tunnel junction made up of two thin ($d \ll \lambda$) metallic films of different superconducting materials with unequal effective electron masses ($m_1^* \neq m_2^*$) and gaps ($\Delta_1 \neq \Delta_2$) separated by an insulating layer, in the presence of an alternating electric field $\mathbf{E} = -(1/c)(\partial\mathbf{A}/\partial t)$ parallel to the plane of the junction and of a slowly varying voltage $V(t)$, applied to the dielectric barrier. To calculate the tunnel current through such an S–I–S' junction we use the well-known tunnel-Hamiltonian method[2,3] with allowance for the different time dependences of the Green's functions (1) and (2) for films with different effective electron masses. Following the known procedure for calculations in the adiabatic approximation as the temperature $T \to 0$, we arrive at the following expression for the tunnel current:

$$I(t) = \mathrm{Im}\left\{I_0(t)\exp\left[\frac{ie^2}{c^2}\left(\frac{1}{m_1^*} - \frac{1}{m_2^*}\right)\int^{t} A^2(t')dt' + 2ie\int^{t} V(t')dt'\right]\right\}, \quad (3)$$

where

$$I_0(t) = \frac{1}{eR}\iint_{-\infty}^{\infty}\frac{d\omega_1 d\omega_2}{(2\pi)^3}\mathrm{Im}[F_1^+(\omega_1)F_2(\omega_2)]\int_{-\infty}^{t}dt'\,\frac{\sin Z_1(t,t')}{Z_1(t,t')}\,\frac{\sin Z_2(t,t')}{Z_2(t,t')}$$

$$\times \exp\left[-i(\omega_1+\omega_2+eV)(t-t') - \frac{ie^2}{2c^2}\left(\frac{1}{m_1^*}-\frac{1}{m_2^*}\right)\int_{t'}^{t} A^2(t_1)\,dt_1\right]. \quad (4)$$

Here

$$Z_i(t,t') = \frac{e}{c}v_{Fi}\int_{t'}^{t} A(t_1)\,dt_1; \quad (5)$$

$$1/R = 4\pi e^2 N_1(0)N_2(0)\langle |T_{\mathbf{kq}}|^2\rangle; \quad (6)$$

$F_i(\omega)$ is the Gor'kov function integrated over the energy, $N_i(0)$ and v_{Fi} are respectively the density of states and the Fermi velocity of the electrons in one of the films $(i=1,2)$, R is the ohmic resistance of the junction in the normal state, $T_{\mathbf{kq}}$ is the matrix element of the tunnel junction, and the symbol $\langle\cdots\rangle$ denotes averaging over all the directions of the vectors $\mathbf{k}$ and $\mathbf{q}$ on the Fermi surface.

In the general case, the time dependence of the current $I(t)$ is rather complicated. However, if $Z_i^2(t,t') \ll 1$, i.e., under the conditions $eE_0v_{Fi} \lesssim \omega^2$, expression (3) reduces to the simpler form

$$I(t) = I_C \sin\left[\frac{e^2E_0^2}{2\omega^2}\left(\frac{1}{m_1^*}-\frac{1}{m_2^*}\right)\left(t-\frac{1}{2\omega}\sin 2\omega t\right) + 2e\int^{t} V(t')\,dt'\right]. \quad (7)$$

Here I_c is the Josephson critical current,[2,3] and its value at $eV \ll \Delta_{1,2}$ is

$$I_C = \frac{1}{eR}\frac{2\Delta_1\Delta_2}{\Delta_1+\Delta_2}K\left(\frac{|\Delta_1-\Delta_2|}{\Delta_1+\Delta_2}\right), \quad (8)$$

where K is a complete elliptic integral of the first kind.

It follows from (7) that the difference between the effective masses of the electrons in the superconducting films on the two sides of the dielectric barrier leads, following application of an alternating electric field $\mathbf{E}(t)$ parallel to the plane of the films, to an additional time dependence not connected with the voltage $V(t)$ on the barrier, of the Josephson current through the S–I–S' junction. This is due to the appearance of a time-oscillating difference between the kinetic energies (chemical potentials) of the Cooper pairs in the films under the influence of the external field $\mathbf{E}(t)$. As shown recently in[4], in two-band superconductors with unequal effective electron masses in the bands, a similar mechanism of acceleration of Cooper pairs in an electric field leads to a redistribution of the electrons among the bands.

It must be emphasized that in the absence of voltage on the junction ($V=0$) the frequency of the fundamental harmonic of the tunnel current $I(t)$, which depends on the amplitude of the alternating electric field E_0 and is equal to $\Omega = e^2E_0^2|m_2^*-m_1^*|/2\omega^2 m_1^* m_2^*$, which is much lower than the frequency ω of the external field. In particular, at $|m_2^*-m_1^*| \sim m_{1,2}^*$ we have the ratio $\Omega/\omega \lesssim \omega/E_F$ (where E_F is the Fermi energy), so that, for example for $\omega \sim 10^{10}$ sec^{-1} and $E_F \sim 1$ eV the frequency is $\Omega \lesssim 10^5$ sec^{-1}.

On the other hand, according to (7), in the given-current regime ($I=\text{const}$), the alternating field $E(t)$ produces on the S–I–S' junction a small voltage

$$\tilde{V}(t) = \frac{eE_0^2}{4\omega^2}\,\frac{(m_2^*-m_1^*)}{m_1^*m_2^*}(1-\cos 2\omega t). \quad (9)$$

This should lead to a shift of the vertical section of the current-voltage characteristic (which corresponds to the stationary Josephson effect in the absence of a field) by an amount $|\tilde{V}| \lesssim \omega^2/eE_F$ which at $\omega \lesssim \Delta \sim 1$ meV and $E_F \sim 1$ eV amounts to $|\tilde{V}| \lesssim 1$ μV.

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