"Evaporation" of black holes and the fundamental length

V. L. Ginzburg

P. N. Lebedev, Physics Institute, USSR Academy of Sciences

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If a fundamental length $l_f \gg l_g \sim 10^{-33}$ cm exists, then the character of the "evaporation" of sufficiently small black holes can be significantly altered in comparison with that considered at $l_f \to 0$. In this connection it becomes possible, in principle, to assess the value of l_f from astronomical data.

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As shown relatively recently,[1] when quantum effects are taken into account, black holes are unstable and are "evaporated"—their mass decreases as a result of emission of photons and other particles. How long will such an evaporation process continue? Within the framework of classical (non-quantum) general relativity theory, no limitations are seen here, and one can expect complete vanishing of the black hole. On the other hand, there is the question of what happens with the baryon charge of the hole. Most importantly, when the hole reaches a mass $M_g \sim \sqrt{c\hbar/G} \sim 10^{-5}$ g, general relativity theory no longer holds (it suffices to state that the quantum fluctuations of the metric are in this case of the order of unity; see, for example[2,3]). One might therefore assume that the evolution of black holes terminates at the stage of a particle with mass $M \sim M_g$ ("maximons"[4]). This conclusion, just as the entire conventional analysis of the evolution of a black hole, is based, however, on the assumption that there is no fundamental length l_f, i.e., there is no characteristic distance (larger than the gravitational length $l_g \sim \sqrt{G\hbar/c^3} \sim 10^{-33}$ cm) at which the existing spatial representations already "break down" (we may deal with some quantization of space, etc.) The results of high-energy physics offer evidence that $l_f \lesssim 10^{-16}$ cm, and thus there is still a colossal gap between the possible values of l_f and the length l_g.

If a length $l_f > l_g$ actually exists, then it might be assumed that the validity of general relativity theory is also restricted to this length in the sense, for example, that the behavior of a body with gravitational radius $r_g = 2GM/c^2 \approx 3\times 10^5 \, (M/M_\odot)$ cm at $r_g \lesssim l_f$ is no longer described by general relativity theory. Similarly, deviations from GRT should appear also near singularities (in particular, near a cosmological singularity) characterized by a scale l_f. But it seems that these latter deviations or changes are extremely difficult to take into account and to observe. To the contrary, it is quite possible for the last stage of evaporation of black holes to be determined most essentially by the value of l_f.

The most natural assumption is that the evaporation (evolution) of the black hole stops at $r_g \sim l_g$, whence $M_f = c^2 r_g/2G \sim c^2 l_f/G \sim 10^{28} l_f \lesssim 10^{12}$ g (at $l_f \lesssim 10^{-16}$ cm). The temperature of the black hole (within the framework of GRT)[1,5,6] is $T = \hbar c^3/8\pi GkM \sim 10^{26} M^{-1}$ K (the mass M is in grams). If the temperature is not bounded because of the existence of some "maximum temperature" than at $l_f \sim 10^{-17}$ cm we have $T_f \sim 10^{15}$ K $\sim 10^{11}$ eV, whereas a mass M_g corresponds to a temperature $T_g \sim 10^{27}$ eV. Thus, the existence of a fundamental length can radically influence the behavior of black holes during the late stages of their evolution and, in particular, the emission spectrum of holes in the region of high energy (and also, if account is taken of the cosmological red shift, in the softer part of the spectra).[5,6]

The assumption that evaporation stops at $r_g \sim l_f$ is, of course, only a hypothesis. A stronger influence of the length l_f is also quite conceivable. Thus, from the constants c, $\hbar$, and l_f we can make up an expression for the density $\rho_f \sim \hbar/cl_f^4 \sim 3\times 10^{26} \, (10^{-16}/l_f)^4$ g/cm^3, whereas $\rho_g \sim c^5/G^2\hbar \sim \hbar/cl_g^4 \sim 10^{94}$ g/cm^3, and according to the limitation used above we have $\tilde{\rho}_f \sim M_f/l_f^3 \sim c^2/Gl_f^2 \gtrsim 10^{60}$ g/cm^3. If it is assumed that the presence of the fundamental length limits the density of matter to a value ρ_f (to be sure, we cannot advance any justification for this assumption other than dimensionality considerations), then at $l_f \gg l_g$ the evaporation of black holes having an admissible mass would in general be unobservable. In fact, at $l_f \sim 10^{-17}$ cm we have $\rho_f \sim 3\times 10^{30}$ g/cm^3 and the mass M of a body with radius $r_g = 2GM/c^2 \sim G\rho_f r_g^3/c^2$ is of the order of 10^{27} g; during the cosmological time $t \sim 10^{18}$ sec this mass would practically not be evaporated at all, and its temperature would be 0.1 K.

The last stages of the evolution (explosion) of black holes cannot be observed simply because there are no corresponding relict holes. But if such explosions could be observed, then their character would apparently be quite sensitive to the value of the fundamental length l_f. It seems to us that in spite of the obvious difficulties, the astronomic approach to an estimate of l_f is worthy of attention, all the more since the prospects of using physical methods (at $l_f < 10^{-17}$) for this purpose are not regarded as bright.

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