Influence of finiteness effects on the character of pion condensation in atomic nuclei

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We calculate the values of the critical parameters which determine the π-condensate instability, for a number of specific nuclei in the range from O^{16} to 114^{298}. It is shown that in light nuclei the stability is first violated for the mode $J^{\pi}=0^-$, and in heavy nuclei all the negative-parity modes $J^{\pi}=0^-, 2^-, 4^-, \ldots$ are condensed simultaneously.

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The question of π condensation in nuclear matter and in atomic nuclei has been intensively discussed in recent years.[1,2]

The quasiparticle-interaction parameters are not known sufficiently well to be able to state that a π condensate exists in a real atomic nucleus. However, even first estimates by Migdal[1] have shown that the situation in nuclei is close to critical, i.e., either the π condensate is already present in them, or else it must arise following a relatively small increase of the density of nuclear matter (for example, in the collision of heavy ions with energy on the order of several hundred MeV per nucleon).

It is therefore of interest to ascertain the character of the π condensation in finite nuclei.

The instability of nuclear matter, which is responsible for the π condensation, sets in at wave vectors k on the order of the Fermi momentum p_F. Therefore in sufficiently large nuclei the π condensation should occur approximately in the same manner as in an infinite system, and the finiteness effects should be significant only near the boundary of the nucleus. This is precisely the situation analyzed in[3]. The question is, however, whether the real nuclei are sufficiently large for such a situation to be realized. It is also of interest to determine the character of π condensation in light nuclei.

To answer all these questions it is necessary to consider the conditions for stability with respect to a formation of π condensate in particular nuclei. These conditions were formulated in[1] for infinite matter by analyzing the D function of the pion in nuclear matter. For finite nuclei, it is more convenient to study the natural frequencies of the equation for the effective field V having a symmetry $\sim \sigma_\alpha \tau_\beta$ [4]:

$$V(\mathbf{r}_1, \omega) = \sigma_\alpha \tau_\beta - \int \gamma(\mathbf{r}_1, \mathbf{r})\, \Phi(\mathbf{r}, \mathbf{r}_2, \omega)\, V(\mathbf{r}_2, \omega)\, d^3r\, d^3r_2 \qquad (1)$$

γ is the amplitude of the quasiparticle-interaction and is irreducible in the particle-hole channel. For our purposes it suffices to approximate γ in the momentum representation by means of a constant γ_0 which is independent of the momenta and by means of one-pion exchange amplitude γ_π, which depends on the momentum transfer k:

$$\gamma = \gamma_0 + \gamma_\pi \,, \qquad (2)$$

$$\gamma_0 = {}^1\!/_2 (g^+ + g^- \vec{\tau}_1 \vec{\tau}_2)\, \vec{\sigma}_1 \vec{\sigma}_2 , \qquad (3)$$

$$\gamma_\pi = - \frac{1.4(1-2\zeta_s)^2 (\vec{\sigma}_1 \mathbf{k})(\vec{\sigma}_2 \mathbf{k})(\vec{\tau}_1 \vec{\tau}_2)}{1 + k^2 - \dfrac{0.9(1-\alpha)k^2}{1+0.23k^2}} \,. \qquad (4)$$

We have used here the units $\hbar = c = m_\pi = 1$, g^+, g^-, α and ζ_s are constants of the theory, with ζ_s determining the renormalization of the axial vertex and α determining the renormalization of the vertex for the production for the Δ isobar in nuclei. The condition of stability with respect to π condensation clears that the square of the natural frequencies ω_0^2 of Eq. (1) be positive.

In certain nuclei, the natural quantum numbers that should be used when solving Eq. (1) are the angular momentum and parity J^π. Equation (1) splits into systems for $J^\pi = 0^-, 1^+, 2^-, 3^+, \ldots$ etc. Each of them determines a set of critical parameters for a given J^π. The influence of the constant g^+ on the eigenvalues of (1) is

J^{π} \ Nucleus	O^{16}	Ca^{40}	Sn^{114}	Pb^{208}	114^{298}
0^-	0.80	1.07	0.99	0.98	0.96
1^+	0.55	0.69	0.88	0.80	0.82
2^-	0.70	0.91	0.98	0.96	0.94
3^+	0.00	0.60	0.89	0.81	0.88
4^-	1)	0.72	0.90	0.98	0.94
5^+	–	0.13	0.75	0.87	0.82
6^-	–	–	0.79	0.95	0.94
7^+	–	–	0.43	0.79	0.84
8^-	–	–	–	0.79	0.91

1)We note that it is meaningful to consider the stability conditions only for $J \lesssim A^{1/3}$.

negligible (estimated at $\sim 0.1(N-Z)/A)^2$, which is $\sim 0.4\%$, for example, for the Pb^{208} nucleus), and therefore this is a three-parameter problem.

The critical values $(g^-_{cr})_{J^\pi}$ at given α and ζ_s were obtained for a number of spherical nuclei in the range from 0^{16} to 114^{298}. Equation (1) was solved in the coordinate representation. Details of the calculation scheme and more detailed results are reported in a separate article.[5] We note here only that the standard technique for solving Eq. (1), based on the use of the representation of the quasiparticle wave eigenfunctions $\zeta_\lambda(r)$ cannot be used here, since it inevitably involves a summation over the states λ.

The table lists the result of the calculation of the critical values of the constants $(g^-_{cr})_{J^\pi}$ for the realistic parameter values $\zeta_s = 0.1$ and $\alpha = 0$. We see that in light and heavy nuclei two essentially different situations are realized. Assume that we are able to decrease the constant g^- "by hand" or, equivalently, increase the density ρ ($\delta\rho/\rho \sim -3\delta g^-/g^-$). Then the first to become unstable in light nuclei is the 0^- mode. Estimates show that the "distance" $\Delta g^- \sim 0.15$ (in Ca^{40}) to the nearest instability point of the mode 2^- is large enough to be able to neglect the influence of the remaining modes on the 0^- instability. Therefore in the case of a second-order phase transition the structure of the condensate should also correspond to the symmetry 0^-.

The situation is different for heavy nuclei, where the instability points are divided into two groups for modes of negative and positive parity. Within each group, the instability points practically coincide, but the distance between groups is $\Delta g^- \approx 0.15$, and the first to condense are the negative-parity modes. The reason for the separation is that in atomic nuclei states in neighboring shells have as a rule opposite parity. This circumstance is a consequence of the quasi-oscillatory form of the nuclear self-consistent shell and should vanish as $A \to \infty$, and A states of both like and unlike parity should be represented to the same degree in a broad rectangular well in the interval $\sim \epsilon_F A^{-1/3}$.

On the other hand, in real heavy nuclei, when g^- is decreased, instability should set in at first practically simultaneously for all the negative-parity modes.

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Erratum: Influence of finiteness effects on the character of pion condensation in atomic nuclei [JETP Lett. 22, 258–259 (20 November 1975)]

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1) On p. 259, left-hand column, 12th line of text "... since it inevitably involves a summation over the states λ." should read "since it inevitably involves cutoff of the summation over the states λ."

2) On the same page and column, 11th and 12th lines of the text, "quasiparticle wave eigenfunctions $\zeta_\lambda(r)$" should read "quasiparticle wave eigenfunctions $\phi_\lambda(r)$."