Collapse of plasma waves near the lower hybrid resonance

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The turbulence of plasma waves with frequencies close to the lower hybrid frequency is investigated analytically and numerically. The existence of collapse—formation of a singularity of the oscillation amplitude within a finite time—is demonstrated.

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1. It can be regarded as established by now that the Langmuir collapse, or the formation of a singularity of the amplitude of Langmuir oscillations in decreased-density zones within a finite time, plays a decisive role in the heating of an isotropic and weakly-magnetized plasma.[1–3] In this article we wish to call attention to the possibility of an analogous phenomenon in an entirely different physical situation, namely when plasma waves are excited near the lower hybrid resonance, with frequencies

$$\omega_{\mathbf{k}} = \omega_L(1 + z^2 + y^2)^{1/2}; \quad z = (M/m)^{1/2}\cos\theta;$$

$$\omega_L^2 = \frac{\omega_H^2\omega_{pi}^2}{\omega_p^2+\omega_H^2}; \quad y^2 = k^2R^2 = \begin{cases} 3k^2\dfrac{T_i}{T_e}r_d^2 \ (\omega_H \gg \omega_p), \\ \left(\dfrac{3}{4} + 3\dfrac{T_i}{T_e}\right)k^2r_H^2(\omega_H \ll \omega_p) \end{cases}. \quad (1)$$

Here ω_H and ω_p are the cyclotron and plasma frequencies of the electrons, and θ is the angle between the wave vector and the magnetic field. An experimental confirmation of the effect considered below is apparently provided by recent experiments[4] in which the formation of oscillation-localization regions stretching along the magnetic field and their self-contraction have been observed.

2. We call attention to the similarity between (1) and the law of dispersion of Langmuir waves. It follows from this that most nonlinear mechanisms lead, within the framework of weak turbulence, to a transfer of energy into the region of small z and k, where a plasma-wave condensate analogous in its origin to the Langmuir condensate is formed.

Let, for example, the plasma waves be excited parametrically with a frequency ω_0 several times larger than ω_L. The weak-turbulence spectrum has in this case three characteristic regions. In region 1 we have $\omega_0 > \omega_k > \sqrt{2}\omega_L$, and the main nonlinear process is either interaction of the plasmons with the ion sound (if $T_e \gg T_i$) or induced scattering by ions ($T_e \sim T_i$). Since obviously $z \gg y$ in this case, the nonlinear energy redistribution is only with respect to angle; as shown in[5], the oscillations are then concentrated in the region of small wavelength, such that $y \approx y_0 = k_0R$ (k_0 is the characteristic wave vector for which the Landau damping of the plasmons is still small). Region 1 is filled with oscillations if

$$\gamma_{\text{nonst}}/\gamma_{\text{damp}} \sim \omega_0/\omega_L y_0. \quad (2)$$

At large excesses, oscillations are excited in region 2, namely $1 > z^2 > y_0^2$. In this region there is a significant change in the character of the nonlinear processes. As shown in[6], principal among these processes, regardless of the temperature ratio, is induced scattering by the electrons.[1)] The nonlinear energy redistribution is again with respect to angle, and the short-wave character of the turbulence is likewise preserved.

Let us consider, finally region 3 of the angles, $z^2 < y_0^2$. As seen from (1), the dependence $\omega = \omega(y)$ is quite significant here. It leads to a change in the direction of the spectral redistribution, whereby both the angles and the absolute values of the wave vectors are altered. We note now that in region 3 the change of frequency (and consequently of the energy) of the plasmons as a result of the nonlinear effects is small, and their total number, as is well known, is conserved in the scattering process. Therefore, more energy is concentrated in the region of small k and z, which leads in turn to instability of the condensate. This instability can no longer be described by assuming the wave turbulence to be weak. Let us ascertain the region of applicability of the weak-turbulence description. To this end we compare the characteristic change of frequency due to the nonlinear interaction and the dispersion term. The condensate-instability growth rate is[6]

$$\gamma_{ne} \sim \omega_L \frac{W}{nT_e}\frac{M}{m}\frac{\omega_p^2}{(\omega_p^2+\omega_H^2)} \quad (3)$$

(W is the oscillation energy). From (1) and (3) we have

$$\frac{W}{nT_e}\frac{M}{m} \ll k^2R^2\left(1+\frac{\omega_H^2}{\omega_p^2}\right). \quad (4)$$

The condition for the onset of the strong instability, which is the inverse of (4), is equivalent to the requirement $\gamma_{\text{inst}}/\gamma_{\text{damp}} \gg \omega_0/\omega_L y_0$; it can be easily satisfied in experiments on parametric plasma heating.

3. The dynamic equation describing the evolution of strongly-nonlinear spectra of plasma waves can be obtained by a method close to that used by Zakharov to describe an isotropic plasma[1]

$$\Delta\left(i\psi_t + \frac{\omega_L}{2}R^2\Delta\psi\right) - \frac{\omega_L}{2}\frac{M}{m}\Delta_z\psi - q\,\mathrm{div}([\vec\nabla\psi\times\vec\nabla\psi^*]_z[\mathbf{h}\times\vec\nabla\psi]) = 0;$$

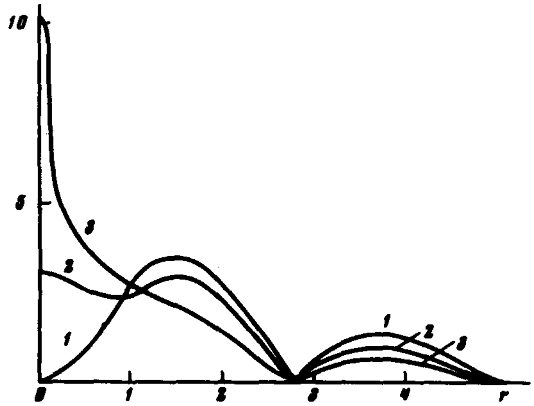

FIG. 1. The function $W(r)$ for the first mode ($m=1$) at the instants of time $t=0$ (1), 0.15 (2), and 0.4 (3); ($H=-8.5$).

$$q = e^2\omega_p^2/2m\omega_L(T_e + T_i)(\omega_p^2 + \omega_H^2);\quad \mathbf{h} = \mathbf{H}_0/H_0. \tag{5}$$

Here ψ is the temporal envelope of the electrostatic potential $\phi_{e1} = \psi\exp[i\omega_L t] + \text{c.c.}$ Equation (5) is valid at sufficiently low intensities

$$\frac{W}{nT_e} << \frac{m}{M}(1 + \omega_p^2/\omega_H^2) \tag{6}$$

so long as the instability growth rate is $\gamma_{ne} \ll k_z v_{Te}$. A more detailed investigation shows that during the course of the instability development it is essential that the relation $\psi - \gamma \sim z$ between the longitudinal and transverse variation scales be satisfied, or equivalently,

$$l_{\parallel}/l_{\perp} \sim (M/m)^{1/2}kR >> 1. \tag{7}$$

When (7) is taken into account, Eq. (5) becomes two-dimensional and acquires, in dimensionless variables, the form

$$\Delta(i\psi_t + \Delta\psi) - 2\,\text{div}([\vec\nabla\psi \times \vec\nabla\psi^*]_z[\mathbf{h}\times\vec\nabla\psi]) = 0. \tag{8}$$

In analogy with the equations for the Langmuir collapse, [1,8] Eq. (8) can be written in Hamiltonian form (as can, incidentally, also (5))

$$\Delta i\psi_t + \frac{\delta H}{\delta\psi^*} = 0;\quad H = \int(|\Delta\psi|^2 + [\vec\nabla\psi\times\vec\nabla\psi^*]_z^2)d\mathbf{r}. \tag{9}$$

From this fact follows directly the conservation of the Hamiltonian H. There is also an integral of the motion $N = \int|\vec\nabla\psi|^2 d\mathbf{r}$, which has the meaning of the total number of plasmons.

In spite of the noted similarity, the question of the possibility of a collapse and its criteria are much more complicated within the framework of (8) than for an isotropic plasma (in the latter case the sufficient condition for the formation of the singularity is that the Hamiltonian be negative). Definite information on the behavior of the solutions of (8) can be obtained by investigating its stationary states ($\psi(t) \propto \exp(i\lambda^2 t)$). It is easy to show that for such solutions we have $H=0$. Thus, an arbitrary initial distribution never reaches a stationary value as a result of the evolution. On the other hand it is obvious that if $H<0$ then $\psi(r)$ cannot spread out without limit, as would be the result of diffraction ($H>0$).

We note that the solutions of (8) cannot be radially symmetrical, owing to the vanishing of its nonlinear part. However, we can seek also a solution in the form $\psi = \Phi(r,t)\exp(im\phi)$ ($m = 1, 2, \ldots$). For the function Φ we obtain the one-dimensional equation

$$\hat K(i\Phi_t + \frac{1}{r}\hat K\Phi) + 2m^2\Phi\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}|\Phi|^2 = 0;\quad \hat K = \frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{m^2}{r}. \tag{10}$$

The existence and the character of the collapse can be assessed from the behavior of the individual modes $\Phi_m(r,t)$, the greatest interest being attached to the first of these modes.

4. Equation (10) was numerically simulated with a computer using an implicit Krank–Nicholson difference scheme of second order accuracy in time and in r, in which the order of the approximation is retained near $r=0$.

The numerical experiment has convincingly demonstrated the presence of a collapse. Its necessary condition is that the Hamiltonian be negative. The character of the formation of the singularity in the solution is clearly seen from Figs. 1 and 2. The calculation accuracy was monitored against the conservation of the Hamiltonian; the calculation was stopped when the maximum deviation ($H - H_0$) amounted to (5–7%).

5. The presence of collapse can exert an appreciable influence on the character of the plasma heating near the lower hybrid resonance. It is very important in this case to ascertain the mechanism whereby the energy is dissipated in the collapse.

In our case, the most powerful stabilizing factor is

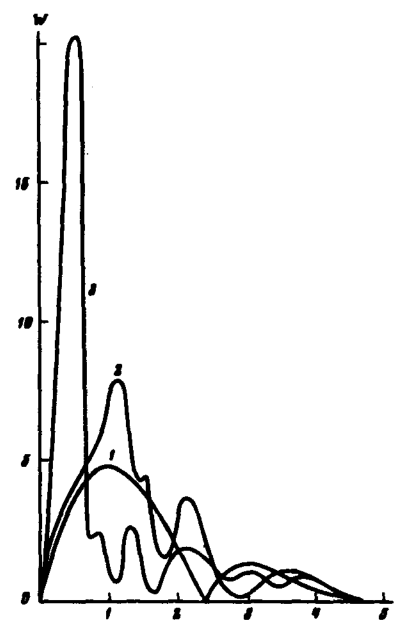

FIG. 2. The function $W(r)$ ($m=2$) at the instants of time $t=0$ (1), 0.125 (2) and 0.19 (3); ($H=-575$).

the Landau damping of the plasma waves. Indeed, it can be shown[6] that even at relatively low intensity (the criterion inverse to (6)) the transverse dimension of the localization region becomes so small that oscillations are excited in the region $kr_H \sim 1$ or $kr_d \sim 1$, where it is necessary to take the quasilinear effects into account. It is quite natural to expect a large number of fast particles to appear in this case.

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1) A similar result was obtained in[7].

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