Superradiance induced in a medium by a fast charged particle and a coherent optical pulse

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It is shown theoretically that radiation of the "Cerenkov" type is produced in a homogeneous medium after the passage of a fast charged particle whose velocity is lower than the phase velocity of the light in the medium. The radiation is produced after the passage of a coherent optical pulse.

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Can a fast charged particle moving in a certain homogeneous medium with velocity lower than that of the light in the medium produce Cerenkov radiation? It is shown below that radiation of this type is always produced if a coherent optical pulse of ultrashort duration ($\sim 10^{-10}$–10^{-11} sec) is made to pass through the medium in the wake of the particle. This pulse produces a special superradiant state, the decay of which is realized in the form of coherent radiation. If τ is the time interval between the passing particle and the exciting light pulse, then the "Cerenkov" cone is produced at the instant 2τ, and its angle depends on the particle velocity and on the propagation velocity of the optical pulse (formula (8))

The proposed effect has a close analogy with the photon echo phenomenon, which was recently investigated intensively in gases and solids.[1] Photon echo is coherent optical radiation emitted by a medium through which two exciting pulses pass in succession, and the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ of the exciting pulses and the wave vector $\mathbf{k}$ of the photon-echo pulse are connected by the relation[2]

$$\mathbf{k} = 2\mathbf{k}_2 - \mathbf{k}_1. \qquad (1)$$

The effect considered in this article is in essence photon echo in which the place of the electromagnetic field of the first pulse is taken by the field of the charged particle. This is confirmed also by a more exact analysis. Let the trajectory of a particle with charge Ze be a straight line and let its velocity v be constant. Assume that at a certain distance from the trajectory (at the impact parameter ρ) there is an immobile atom. If the atom is initially in the ground state, then after the fast passage of the charged particle it turns out to be in a coherent quantum state $|\psi\rangle = |0\rangle + \sum_m a_m |m\rangle$, where m numbers the excited states. For the known electric field of a relativistic charged particle we obtain on the basis of perturbation theory the following expression for the amplitudes a_m:

$$a_m = i\theta_m \exp(ik_m z), \qquad (2)$$

$$\theta_m(\rho) = -\frac{d_{m0}}{\hbar v}\int_{-\infty}^{+\infty} E_\perp(x)\exp(ik_m x)\,dx = 2\frac{d_{m0}Ze}{\hbar v}\frac{\partial}{\partial\rho}K_0\left(\frac{\omega_{m0}\rho}{v\gamma}\right). \qquad (3)$$

Here $k_m = v^{-1}\omega_{m0}$; ω_{m0} and d_{m0} are the frequency and the dipole moment of the transition between the states $|m\rangle$ and $|0\rangle$, $E_\perp = Ze\gamma^{-2}(x^2+\gamma^{-2}\rho^2)^{-3/2}$ is the particle electric-field component at the point where the atom is located (normal to the trajectory), x is the coordinate along the trajectory, $\gamma + (1 - v^2/c^2)^{-1/2}$, and K_0 is a modified Bessel function of the second kind. We emphasize that the phases of the amplitudes (2) are preserved during a certain time T_2 (the time of the phase memory), which can vary in a wide range (from 10^{-12} to 10^{-6} sec).

We assume furthermore that the concentration of the atoms is N and that their transition frequencies have a scatter with half-width $\Delta\omega_0$ about a certain frequency ω_0 in accord with the dispersion law (Lorentz line shape). Then the average dipole moment of the medium at each transition frequency takes the form

$$P_m = iNd_{m0}\theta_m(\rho)\exp\left\{-\frac{1}{T_2^*}\left|t - \frac{z}{v}\right| - i\omega_0\left(t - \frac{z}{v}\right)\right\} + \text{c. c.}, \qquad (4)$$

where $T_2^* = 1/\Delta\omega$ describes the damping of the dipole moment as a result of the frequency detuning. It is seen from (4) that no coherent radiation is produced if $v < c$.

Assume now that a light pulse passes through the medium at the instant of time $t = \tau$ and its frequency ω coincides with one of the transition frequencies ω_{m0}, while its duration is $\Delta\tau \ll T_2$. The resonant interaction leads to a large change in the amplitudes a_m at the selected transition and as a result the average dipole moment is completely changed. Within the framework of the described method we obtain for P_m

$$P_m = -iNd_{m0}\theta_m(\rho)\sin^2\left(\frac{\theta^2}{2}\right) \times \exp\left\{-\frac{1}{T_2^*}\left|t - 2\tau - \frac{z}{v}\right| + ik_z z - i\omega t\right\} + \text{c. c.} \qquad (5)$$

$$\mathbf{k} = \frac{\omega}{c}(2\mathbf{n}_2 - \mathbf{n}_1), \qquad (6)$$

where $\theta_2 = d_{m0}\hbar^{-1}\int_0^{\Delta\tau} dt' E(t')$ is the so-called "turning angle" in a two-level system and is due to the optical pulse with field amplitude $E(t')$, while $\mathbf{n}_1$ and $\mathbf{n}_2$ are the vectors of the particle-motion and light-pulse directions. It is seen from (5) that P_m is maximal at $t = 2\tau$, in full analogy with the photon-echo phenomenon, and an analysis of the phase factor shows that coherent radia-

... produced if the condition $c/\varepsilon < v < c$ is satisfied.

Substituting the polarization (5) in Maxwell's equations and solving the latter by the well known method,[3] we can obtain the components of the electromagnetic field and the energy W radiated by the particle through the surface of a cylinder of length l and with axis along the trajectory. The calculation can be carried through to conclusion and its result is

$$\frac{dW}{dl} = 64\pi^3 \sin^2\left(\frac{\theta_2}{2}\right)\cos^2\phi\,[Ze\gamma N d_{m0}^2 c^{-1}\hbar^{-1}]^2 \omega T_2^* e^{-2\tau/T_2},$$

$$\cos\phi = 2\cos\chi - \frac{c}{v}. \quad (7)$$

The angles ϕ and χ are those between the vectors $\mathbf{k}$, $\mathbf{n}_1$ and $\mathbf{n}_2$, $\mathbf{n}_1$, respectively. For a Gaussian frequency distribution it is necessary to replace T_2^* by $T_2^* 2\sqrt{\pi}$. It follows from (4), (6), and (7) that at $\chi \sim 1$ and at $2\tau/T_2 \sim 1$ the energy W is of the same order of magnitude as for Cerenkov radiation of a particle in a gas (at $v > c$). However the requirement with respect to the threshold energy is much less stringent in our case. For an electron, for example, it amounts to 25 eV.

We note also that by proper choice of the optical-pulse parameters the medium can be made amplifying (if $\theta_2 \approx \pi$), and this leads to an additional increase of the radiation energy. The use of two-photon excitation makes it possible to invert the medium at large depths.

Finally, a similar effect is possible if the particle is followed by several optical pulses in different directions; this corresponds mainly to "stimulated" and "reconstructed" echo signals.[4] This lowers appreciably the threshold energy in comparison with even the case considered here. It is interesting that the "echo" effect can be produced in principle by two successively moving charged particles (or particle bunches). In this case coherent radiation is produced immediately at all the transition frequencies.

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