Possibility of the existence of X mesons of a new type

N. A. Voronov, I. Yu.Kobzarev, and N. B. Konyukhova

Institute of Theoretical and Experimental Physics
(Submitted October 17, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **22**, No. 11, 590–594 (5 December 1975)

It is shown by a numerical experiment for a scalar field with spontaneous symmetry breaking that three-dimensional soliton-like systems exist. Such objects could be revealed in experiment as heavy long-lived mesons.

PACS numbers: 14.40.—n, 11.10.Qr

Many authors have discussed the hypothesis according to which the classical solutions of relativistic equations with degenerate vacuum can describe hadronic states.[1–3] In particular, Shapiro has proposed that solutions of this type describe mesons.[4] For the one-dimensional case, there are known both fully stable (soliton) and quasi-stable (soliton-like) solutions.[5–7] In a real three-dimensional case, however, neither stable nor quasi-stable solutions have been obtained so far, with the exception of the solutions of the monopole type, which do not go over into the ordinary vacuum at infinity.[2] We describe here the results of a numerical experiment that shows that in the case of a field with two vacuums there exist pulsating quasi-stable three-dimensional solutions with the usual vacuum at infinity; their properties differ significantly from the properties of the one-dimensional solutions.[5–7] Such objects can correspond to heavy mesons with zero spin.

We consider the equation

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = -4\lambda^2 u(u^2 - \eta^2), \tag{1}$$

which has two vacuum solutions $u = \pm\eta$ with zero energy. We are interested in a solution of the bubble type,[8] which is close to one of the vacuums everywhere except for a finite singly-connected region, inside of which it is close to the second vacuum. The energy of the bubble is concentrated in the transition layer (wall) between the vacuums.

The collapse of a spherical bubble was considered in[8] in the approximation $R \gg l$, where R is the radius of the bubble and l is the wall thickness ($l \sim 1/\lambda\eta$). It is easy to show that in this approximation the equation of motion of the wall is

$$\frac{d^2R}{dt^2} + \frac{2}{R}\left[1 - \left(\frac{dR}{dt}\right)^2\right] = 0. \tag{2}$$

Solving (2), we obtain

$$R = R_0\, cn\left(\frac{\sqrt{2}\,t}{R_0}, \frac{1}{2}\right), \tag{3}$$

where $cn(x, 1/2)$ is the elliptic cosine with modulus $k^2 = 1/2$. The expression (3) described satisfactorily the contraction or the expansion of the bubble, but since it is not valid at $R \sim l$, Eq. (3) cannot describe the transition from contraction to expansion. To ascertain the possibility of such a transition, which would lead to a pulsating behavior of R, we turn directly to Eq. (1).

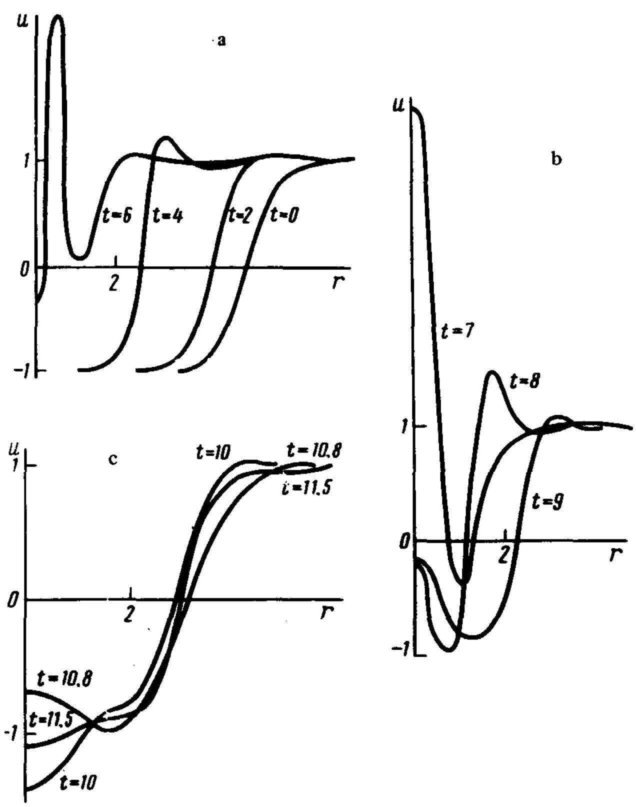
The problem was solved numerically by the straight-line method, i.e., by reducing the partial-differential equations (1) to a system of ordinary differential equations specified on the straight lines $t = nh$ ($n = 1, 2, \ldots, h$ is a constant chosen to be the time interval). The transfer of the boundary conditions from the singular points $r = 0$ and $r = \infty$ to the finite points r_0 and r_∞ was realized for the entire system as a whole using the method developed in[9]. To solve the nonlinear boundary-value problem, a converging interactive process, analogous to the one described in[10], was used on each line. The linear boundary-value problem was calculated in each iteration by the Abramov stable run-through method.[11] The integral of the energy and the fluxes at the points r_0 and r_∞ were calculated on each straight line.

By means of similarity transformations, Eq. (1) reduces to the case $\lambda = 1$, $\eta = 1$, which will be discussed from now on. We choose the following initial conditions:

$$u(r, 0) = \mathrm{th}\sqrt{2}(r - R_0), \qquad u_t(r, 0) = 0.$$

The results of numerical calculations with $R_0 = 5$ are shown in the figure. At $6 < t < 8$ the function u has an overshoot in the vicinity of $r = 0$ with $u_{max} \approx 3.9$, after which an expanding bubble appears again. At $t = 10.8$ it stops again. We note that the function u vanishes at that instant of time at the point $R_1 \approx 3$. The backward motion of the bubble to the center is noticeable already at $t = 11.5$.

The large difference between R_1 and R_0 (the initial radius) is attributed to the loss of the energy integral. The energy, equal to $E_0 = 6 \times 10^2$ at the initial instant, is quite well preserved up to $t \sim 4$, and then decreases rapidly at $4 < t < 9$ to $E_1 = 2.4 \times 10^2$, after which it again stabilizes to the end of the calculation. We note further that $E_0/E_1 \approx R_0^2/R_1^2$, from which so large a decrease of R follows. It is important that the decrease of E is not due to radiation; the total radiated energy is equal to 9×10^{-2}. As seen from the diagrams shown in the figure, the energy integral is lost at large $\partial u/\partial t$, this being due to the low accuracy with which the time derivatives are calculated in the program (accurate to the time interval h). We also verified the dependence of the solution



on r_∞: an increase of r_∞ by 1.5 times (from 8 to 12) did not affect the function u at all. Thus, the existence of pulsations can be regarded as established.

The behavior of the solution at $6 < t < 8$ can be qualitatively described with the aid of the following reasoning. We dissect the bubble along the diameter and consider its opposite walls. We assume that they pass freely through each other, retaining the shape at which they have approached the center. Then it can be readily seen that the function u has a plateau at the level $u = 3$–4. This state has a constant energy density ϵ, and therefore the potential energy of this state will increase like L^3, where L is the distance by which the walls became separated after they passed through each other; it is clear that the walls must ultimately stop, and then reverse their motion. If it is assumed that the radiation is small, then the maximum value $L_{\max}$ is determined from the equation $\mu R_0^2 = \mu L^2 + (\epsilon/3)L^3$, where $\mu = (4\sqrt{2}/3)\lambda\eta^3$ is the surface energy density of the bubble; it follows therefore that

$$L \sim R_0 \sqrt[3]{l/R_0}\,. \tag{4}$$

Similar reasoning is valid also in the one-dimensional case.[7] It is important that the three-dimensional solutions considered here, unlike the one-dimensional ones, have no limitations on the system mass. In the solution obtained by us $R_0 \gg l$, but solutions with $R_0 \sim l$ are also possible and correspond to the systems obtained in[5–7].

We note that in theories that have at least three equidistant vacuums (as, e.g., in the Gordon sine equation), the same reasoning leads to the conclusion that the walls jump freely through each other, the transition from contraction to expansion being essentially different. To describe the transition in such theories it is natural to use (3), in which case $R_{\text{observ}} = |R|$.

To estimate the bubble pulsation period T in the theory with spontaneous symmetry breaking we can use (3), from which we get $T \sim 3R_0$. In this case we neglect the reflection time, which is small by virtue of (4). Unfortunately, the loss of the energy integral does not make it possible to find the bubble lifetime τ, but judging from the flux of the radiated energy it is larger than T by many orders of magnitude. The bubble radius is connected with its mass by the relation $M = (16\pi\sqrt{2}/3) \times\lambda\eta^3 R_0^2$, from which it follows that $T \approx 0.6\sqrt{M/\lambda\eta^3}$. Since $\tau \gg T$; the lifetime may turn out to be much larger than nuclear even at $\lambda = 1$ and $\eta = m_p$. We note that quasiclassical quantization of the bubble oscillations yields a mass spectrum with a state density $dn/dE \sim T \sim \sqrt{M}$.

We note in conclusion that if the model of Drell *et al.*[12] is valid, in which the ordinary hadrons are regarded as bubbles with quarks locked into their walls, then the mesons discussed here must inevitably exist.

The authors thank T. A. Belova for great help with the computer calculations, and A. E. Kudryavtsev for useful discussions.

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