Production of soft pions by neutral currents in a neutrino experiment

B. L. Ioffe and A. V. Smilga

Institute of Theoretical and Experimental Physics
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Relations are obtained for the differential cross sections of the deep-inelastic processes $\nu+N\to\nu+\pi(\text{soft})+$ hadrons and $\nu+N\nu+2\pi(\text{soft})+$ hadrons. Measurement of these cross sections yields information on the spatial and isotopic structures of the neutral currents in νN interactions.

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At the present time neither the spatial nor the isotopic structure of the neutral currents in νN $(\bar{\nu}N)$ scattering is clear. One of the possible methods of determining this structure (besides the investigation of definite exclusive channels) is to study the production of soft pions in deep-inelastic $\nu(\bar{\nu})N$ scattering. The investigation of these processes is obviously also of independent interest.

In this paper, on the basis of the PCAC hypothesis, we calculate the cross sections for deep-inelastic scattering of neutrinos (antineutrinos) by nucleons with emission of one or two soft pions due to the interaction of the neutral currents, i.e., the processes $\nu(\bar{\nu})N\to\nu(\bar{\nu})+n\pi(\text{soft})+\text{hadrons}$, with $n=1$ or 2. The main premises and the method of analysis are analogous to those used in[1], where the production of soft pions was investigated in deep-inelastic electroproduction and scattering of neutrinos due to charged currents.

We assume that the weak neutral current $J_{w\mu}$ is a superposition of vector and axial components $J_{w\mu}=\alpha_3V^3-\alpha_3A^3+\alpha_0V^0-\beta_0A^0$, where the indices 0 and 3 corresponds to the isoscalar and isovector contributions. (The normalization is such that $\int V_0^3(x)d^3x=T^3$, where T is the isospin vector.) In the Weinberg model[2] we have $\alpha_3=1$ and $\beta_3=\cos2\theta_W$, where θ_W is the Weinberg angle and α_0 and β_0 take on various values, depending on whether the quarks are assumed to have integer or fractional charges.

Consider the process $\nu+N\to\nu+\pi(\text{soft})+\text{hadrons}$. The use of the PCAC hypothesis yields

$$<\text{hadr.},\pi^+|\,J_{u\mu}\,|\,N> = \frac{1}{f_\pi}\{\,ik_\nu\int d^4x e^{ikx}<\text{hadr.}|\,T\{A_\nu^+(x),J_{w\mu}(0)\}\,|\,N>$$

$$+<\text{hadr.}|\,[Q_5^+(0),J_{w\mu}(0)]\,|\,N>\}\,, \qquad (1)$$

where $f_\pi=0.93\mu_\pi=\sqrt{2}g_Am_N/g_r$, and $Q_3^i(x^0)=\int A_0^i(x)d^3x$. At a pion momentum $k\to0$, only the pole diagrams with axial current, inserted in any final baryon line, contribute to the first term. As shown in[1], at $\mathbf{k}^2/k_0^2\ll1$ in the lab. frame, the contribution of these pole diagrams to the cross section is small. (It is assumed here that the ratio $\mathbf{p}^2/m^2$ for the final baryon in the deep-inelastic eN or νN scattering is small: $p^2/m^2\sim0.1$–0.2, in analogy with the situation, e.g., in πN scattering, where $\mathbf{p}^2/m^2=1/8$. [3]) Calculating the commutator, we obtain

$$<\text{hadr.},\pi^+\,(\text{soft})\,|\,J_{w\mu}\,|\,N> = \frac{1}{f_\pi}<\text{hadr.}|\,\alpha_3A_\mu^+(0)-\beta_3V_\mu^+(0)\,|\,N>. \qquad (2)$$

It is known that from the experimentally measured ratio of the total cross sections of the processes $\bar{\nu}_\mu N\to\mu^++\text{hadrons}$ and $\nu_\mu N\to\mu^-+\text{hadrons}$, namely $\sigma_{\bar{\nu}N}/\sigma_{\nu N}=1/3$,[4] and from the kinematic restrictions on the structure functions $F_{1,2,3}(x)$ of these processes, it follows that

$$F_2(x)=2xF_1(x)=-xF_3(x)\,, \qquad (3)$$

where $x=-q^2/2\nu$ is the scaling variable. The equality (3) means that the interference of V and A is maximal and that the contributions of V and A to $F_{1,2}$ are equal. A similar conclusion can be drawn also in the parton-quark model when account is taken of only the contribution of the valent quarks. Using this circumstance, we can easily express the contribution (2) to the inclusive cross section in terms of the structure functions $F_{1,2,3}(x)$, and obtain

$$\frac{d\sigma(\nu(\bar{\nu})N\to\nu(\bar{\nu})\pi^++\text{hadr.})}{dq^2d\nu dE_+} = \frac{|\mathbf{k}_+|}{(2\pi)^2f_\pi^2}\,\frac{d\sigma(\bar{\nu}N\to\mu^++\text{hadr.})}{dq^2\,d\nu}\left[\gamma_\pm\left(\frac{E}{E'}\right)^2+\gamma_\mp\right], \qquad (4a)$$

where $\gamma_\pm=(\alpha_3\pm\beta_3)^2/4$, k_+ is the π^+-meson momentum, E and E' are the energies of the initial and final neutrinos, $\nu=(E-E')m$, $q^2=-4EE'\sin^2(\theta/2)$, the upper sign in (4a) pertains to the neutrino, and the lower to the antineutrino.

In the Weinberg model we have $\gamma_-=\sin^4\theta_W$ and $\gamma_+=\cos^4\theta_W$. We obtain analogously the formulas

$$\frac{d\sigma(\nu(\bar{\nu})N\to\nu(\bar{\nu})\pi^-+\text{hadr.})}{dq^2\,d\nu dE_-} = \frac{|\mathbf{k}_-|}{(2\pi)^3f_\pi^2}\,\frac{d\sigma(\nu N\to\mu^-+\text{hadr.})}{dq^2d\nu}\left[\gamma_\mp\left(\frac{E'}{E}\right)^2+\gamma_\pm\right]. \qquad (4b)$$

The differential cross section of the reaction with emission of a soft π^0 meson vanishes in our approximation. Measuring it, we can, in principle, assess the character of the pole terms in (1).

We consider now reactions with emission of two pions. In the calculation of the matrix elements of such processes it is convenient, following Weinberg's method,[5] to consider the matrix element for the emission of two axial currents

$$M_{\mu\nu}^{ij}(k_1,k_2) = \int e^{i(k_1x+k_2y)}d^4xd^4y<\text{hadr.}|\,T\{A_\mu^i(x),A_\nu^j(y),J_w(0)\}\,|\,N> \qquad (5)$$

assuming that $\partial_\mu A^i_\mu = 0$ and that the pion mass is zero. Using current algebra, we obtain[1)]

$$k_{1\mu}k_{2\nu}M^{ij}_{\mu\nu}(k_1,k_2) = -\frac{1}{2}<\text{hadr.}|[Q^i_5,[Q^j_5,J_w]]+[Q^j_5,[Q^i_5,J_w]]|N>$$

$$+\frac{1}{2}\epsilon_{ije}(k_2-k_1)_\mu \int e^{i(k_1+k_2)x}d^4x <\text{hadr.}|T\{V^l_\mu(x),\ J_w(0)\}|N>. \quad (6)$$

The second term of the right-hand side of (b) receives as $k_1\to 0$ and $k_2\to 0$ contributions from only those diagrams in which the vector current is inserted in external lines, i.e.,

$$-i\int e^{i(k_1+k_2)x}d^4x <\text{hadr.}|\ T\{V^l_\mu(x),\ J_w(0)\}|N> = \sum_n \frac{c_n p_{n\mu}}{p_n(k_1+k_2)}. \quad (7)$$

It follows therefore that

$$\sum_n c_n = <\text{hadr.}|[Q^l,J_w]|N>,\quad Q^l = \int V^l_0(x)d^3x. \quad (8)$$

We shall now assume that $k_{1\mu}$ and $k_{2\mu}$ tend to zero in such a way that the momentum of each of the pions is small, $|\mathbf{k}_1|/k_{10}\ll 1$ and $|\mathbf{k}_2|/k_{20}\ll 1$. Then the left-hand side of (6) is expressed in terms of the matrix element of interest to us, for the emission of two soft pions in the process $\nu N\to\nu+2\pi+$hadrons (we neglect, for the same reasons as before, the pole terms corresponding to the emission of one pion by the initial and final baryons). Taking (7) and (8) into account, we get after simple transformations

$$M^{ij}_{\pi\pi} = \frac{1}{f^2_\pi}<\text{hadr.}\Big|[Q^i_5,[Q^j_5,J_w]]\frac{k_{10}}{k_{10}+k_{20}}$$

$$+[Q^j_5,[Q^i_5,J_w]]\frac{k_{20}}{k_{10}+k_{20}}\Big|N>. \quad (9)$$

For a real process we have $k_{10}=k_{20}=\mu_\pi$, and consequently

$$M^{ij}_{\pi\pi} = \frac{1}{2f^2_\pi}<\text{hadr.}|[Q^i_5,[Q^j_5,J_w]]+[Q^j_5,[Q^i_5,J_w]]|N>. \quad (10)$$

From (10) we get the following expressions for the differential cross sections of the processes $\nu(\bar\nu)N\to\nu(\bar\nu)+2\pi(\text{soft})+\text{hadrons}$[2)]

$$\frac{d\sigma^{\pi^+\pi^0}_{\nu(\bar\nu)}}{dq^2d\nu dE_+dE_0} = \frac{d\sigma^{\bar\nu}_{ch}}{dq^2d\nu}\frac{1}{2}\left\{\gamma_\mp+\left(\frac{E}{E'}\right)^2\gamma_\pm\right\}\frac{k_+k_0}{(2\pi)^4f^4_\pi}, \quad (11a)$$

$$\frac{d\sigma^{\pi^-\pi^0}_{\nu(\bar\nu)}}{dq^2d\nu dE_-dE_0} = \frac{d\sigma^{\nu}_{ch}}{dq^2d\nu}\frac{1}{2}\left\{\gamma_\pm+\left(\frac{E'}{E}\right)\gamma_\mp\right\}\frac{k_-k_0}{(2\pi)^4f^4_\pi}, \quad (11b)$$

$$\frac{d\sigma^{\pi^+\pi^-}_{\nu(\bar\nu)}}{dq^2d\nu dE_-dE_+} = 4\frac{d\sigma^{\nu(\bar\nu)}_{\text{neutr},T=1}}{dq^2d\nu}\frac{k_+k_-}{(2\pi)^4f^4_\pi}, \quad (11c)$$

where $\sigma^{\nu(\bar\nu)}_{\text{neutr},T=1}$ is the cross section for the scattering of a neutrino (antineutrino) by a nucleon on account of a neutral isovector current.

[1)]A similar analysis was carried out by Weinberg in the case of k_{e_4} decay.[6]

[2)]We take the opportunity to correct an error made by one of us in[1]; the expression given there for the electroproduction cross sections of soft $\pi^\pm\pi^0$ mesons should be decreased by a factor eight.

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