Dynamic description of low-energy πN scattering

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Using a linear $SU(2)\times SU(2)$-invariant Lagrangian, the Padé approximation, and the superpropagator regularization method, we obtain a dynamic description of the low-energy πN scattering without introducing any arbitrary parameters whatever. Analytic expressions are obtained for the parameters of the ρ, σ, 11, and 33 resonances.

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The aggregate of the accumulated data indicates that two approaches are possible to the description of strong interactions of elementary particles at low energies.

The first approach is based on the idea of one particle exchange and can be realized with the aid of linear Lagrangians of the Yukawa type, or with the aid of the pole approximations in the dispersion relations. But this approach calls for the introduction of fields corresponding to each interacting particle, including the exchanged ones. The number of such particles is at present quite large and continues to increase as experimental information is gathered. It is clear that introduction of an ever increasing number of fields contradicts the very spirit of quantum field theory.

The second approach is based on the assumption that the number of exchange particles must be limited. Some particles should be bound states or resonances of other more fundamental particles, i.e., should be of dynamic origin. This approach can be realized with the aid of nonlinear Lagrangians. Such a dynamic description for a number of fundamental processes of strongly interacting particles has not yet been realized to date. In this paper we wish to report an attempt at such a dynamic description of low-energy πN scattering.

From the physical point of view, the most interesting are nonlinear Lagrangians of the chiral type. We therefore use here a nonlinear chiral $SU(2)\times SU(2)$-invariant Lagrangian in an exponential parametrization.[1] As already noted by Lehman,[2] the advantage of this parametrization lies in the fact that it leads to a microcausal theory that is localizable in the Meiman–Jaffe sense. The Lagrangian of the interaction of nucleons and pions, which contributes to the πN scattering, is given by

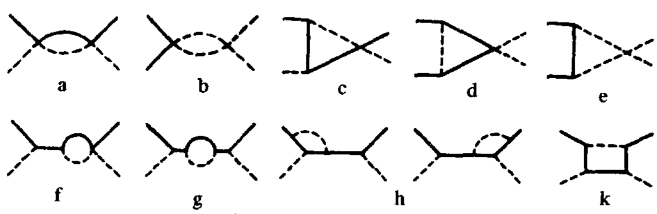
$$L_{\rm int}(x) = \frac{1}{2}[f^2(\vec{\pi}^2)-1][\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi}-(\vec{\pi}\,\partial_\mu\vec{\pi})^2/\vec{\pi}^2]$$
$$-\frac{img_A}{F_\pi}\bar{\psi}\gamma_5\vec{\tau}\,\psi\vec{\pi}\sin Z_1/Z_1 - m\bar{\psi}\psi(\cos Z_1-1)$$
$$+\frac{1}{2}\bar{\psi}\gamma_\mu\vec{\tau}\,\psi\,\vec{\pi}\times\partial^\mu\vec{\pi}[(g_A-1)\sin^2 Z_2-(g_A+1)\sin^2 Z_3]/\vec{\pi}^2, \quad (1)$$

where

$$f(\vec{\pi}^2) = \sin Z/Z, \quad Z=\sqrt{\vec{\pi}^2/F_\pi^2}, \quad Z_1=\sqrt{g_A^2\vec{\pi}^2/F_\pi^2},$$
$$Z_2=\sqrt{(g_A+1)^2\vec{\pi}^2/(4F_\pi^2)}, \quad Z_3=\sqrt{(g_A-1)^2\vec{\pi}^2/(4F_\pi^2)},$$

F_π is the pion weak-decay constant ($F_\pi\approx 92$ MeV), g_A is the renormalized axial-vector coupling constant, and m is the nucleon mass. The pseudovector coupling was excluded from the interaction Lagrangian (1) with the aid of the Foldy–Dyson transformation.

By starting with the Lagrangian (1), we have calculated the amplitude of the πN scattering in the first and second nonvanishing approximations of the perturbation method. In this amplitude, in the chosen approximation, contributions are made, besides diagrams of the tree type, also by the following diagrams (see the figure): The diagram k is finite, and all the others are divergent. The divergences in diagrams c, d, e, f, g, and h are eliminated by the usual renormalization of the charges and masses, while the divergences of diagrams a and b are eliminated with the aid of the superpropagator regularization method proposed by Volkov.[3] Starting from the first and second nonvanishing orders of the perturbation method for the partial amplitudes, we obtain analytic expressions with the aid of the (1, 1) Padé approximation.



The partial s-waves reconstructed in this manner have singularities that are equivalent to the t-channel singularities obtained by allowance for the exchange of the ρ and σ mesons or in the dispersion approach.[4] Comparing the expression for the partial s waves obtained in the two indicated ways, we obtain the following closed expressions for the masses and the coupling constants of the two-pion resonances

$$\frac{m_\rho^2}{m_\pi^2}\approx\frac{\pi}{2f^2(g_A^2-1)}, \quad \frac{m_\sigma^2}{m_\pi^2}\approx\frac{\pi}{f^2},$$
$$\frac{g_{\rho\pi\pi}g_{\rho NN}}{4\pi}\approx\frac{\pi}{2g_A^2(g_A^2-1)}, \quad \frac{g_{\sigma\pi\pi}g_{\sigma NN}}{4\pi}\approx\frac{\pi^2}{2f^2m}, \quad (2)$$

where $f^2=0.88$. At the experimental values $g_A\approx 1.3$ we have $m_\rho^2\approx 30m_\pi^2$, $m_\sigma^2\approx 40m_\pi^2$, $g_{\rho\pi\pi}^2/4\pi\approx 2.6$, $g_{\sigma\pi\pi}g_{\sigma NN}/4\pi\approx 10$. The p_{33} wave obtained in the same manner has a resonance at

$$\omega_{33} \approx \frac{\pi}{2.2mf^2} m_\pi^2, \tag{3}$$

which yields numerically the value $\omega_{33} \approx 2.6m_\pi$ (the experimental value is $\omega_{33} \approx 2m_\pi$). The waves p_{13} and p_{31} have no singularities and turn out to be small. The wave p_{11} calls for the use of additional considerations based on an analysis of the dispersion relations. It turns out that it has a resonance at

$$\omega_{11} \approx \left(\frac{\pi}{mf^2} - \frac{2}{m_\pi}\right) m_\pi^2 \approx 4m_\pi \quad , \tag{4}$$

(the experimental value is $\omega_{11} \approx 3.7m_\pi$). The obtained partial amplitudes describe well the experimental data up to 400 MeV kinetic energy of the pion in the l. s. Thus, by developing the ideas advanced by us in[5], we were able to present the first description of the πN-scattering process without introducing any arbitrary parameters whatever, i.e., to present a dynamic description.

The gist of the dynamic description of the interaction of the elementary particles, based on the use of the Lagrangian formalism, is that all the physical amplitudes are determined by a limited number of fundamental parameters, namely by the values of the masses of the nucleon and the pion (m, m_π) and the coupling constants f^2 and g_A. Thus, for example, the ρ-meson mass, which is determined as the t-channel singularity of the πN-scattering amplitude, is expressed in terms of these initial parameters via expression (2). It therefore becomes unnecessary to introduce fields corresponding to meson and baryon resonances.

These results, together with the results on $\pi\pi$ scattering,[2,6] indicate that the method described above can hopefully be used to obtain a dynamic description of the entire low-energy physics of strong interactions.

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[1]P. Chang and F. Gursey, Phys. Rev. 164, 1752 (1967); F. Gursey, ITP-Budapest Report 249, 1968.
[2]H. Lehman and H. Trute, Nucl. Phys. 52, 280 (1973).
[3]M. K. Volkov, El. Chast. At. Yad. 2, 33 (1971) [Sov. J. Part. Nuclei 2, No. 1, 22 (1971)].
[4]V. V. Serebryakov and D. V. Shirkov, *ibid*. 1, 171 (1970) [1, No. 1, 106 (1970)].
[5]M. I. Gaisak and V. I. Lend'el, Pis'ma Zh. Eksp. Teor. Fiz. 19, 772 (1974) [JETP Lett. 19, 396 (1974)].
[6]M. K. Volkov and V. N. Pervushin, Yad. Fiz. 20, 762 (1974) [Sov. J. Nuc. Phys. 20, 408 (1975)].