where ΔT is the temperature difference between the cylinder axis and its wall, and κ is the thermal conductivity coefficient of the gas. We assume that a mixture of xenon and helium is used to increase the thermal conductivity, with $p_{He} \gg p_{Xe}$, so that $\kappa \simeq \kappa_{He}$ ($\kappa_{He} \simeq 4 \times 10^{-4}$ cal-deg^{-1}sec^{-1}cm^{-1}). The capillary diameter at $\Delta T \simeq 200°K$, $K \simeq 0.05$ cm^{-1} ($W\varepsilon_0 \simeq 5 \times 10^5$ W-cm^{-3}), and $\eta = 0.5$ is then 0.05 mm.

We note that the presence of helium changes the time τ and leads possibly to other effects, for example to the appearance of the $HeXe\,^3\Sigma^+$ molecule and of a new emission band; relations (2) will remain in force, however.

The development of a molecular xenon laser is perfectly realistic. The use of other noble gases for this purpose may encounter serious difficulties due to the rapid increase of τ_{rad} on going to lighter atoms. In principle, one cannot exclude the possibility of lasing with molecules of the type ArXe, KrXe, etc., produced in collisions $Ar\,^3P + Xe\,^1S_0 + M$, $Kr\,^3P + Xe\,^1S_0 + M$, etc. The time τ_{rad} of such molecules may be shorter than the time of the radiative transition $^3P - {}^1S_0$ in atoms, since the presence of xenon should contribute to violation of the Wigner rule. Several elementary processes that lead to the excitation of noble-gas atoms and to the formation of diatomic molecules by passage of an electron beam through a mixture of argon and krypton or xenon were investigated in [6].

We can expect lasers of this type to have high efficiency and power, and to be tunable in a relatively wide frequency range ($\sim$5000 cm^{-1}).

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OSCILLATIONS OF A TYPE-II SUPERCONDUCTOR IN A MAGNETIC FIELD

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Andronikashvili, Chigvinadze, et al. [1 - 4] have tested in their experiments an original currentless procedure for the investigation of pinning forces and vortex interactions in type-II superconductors. In these experiments, a superconducting cylinder in the mixed state [5] executes small axial oscillations in a magnetic field perpendicular to the cylinder axis, and the dependence of the frequency and the damping decrement of the oscillations on the field, temperature, sample purity, and other factors is measured (Fig. 1, where the curves are taken from [4]).

The author has advanced elementary theoretical considerations that explain the unusual character of these relations (the damping in the superconducting state is larger than in the normal state!). The increase of the oscillation frequency in the mixed state (Fig. 1) is evidence of the presence of a "frozen-in" magnetic field in the sample; this field is produced by Abrikosov vortices pinned by various defects [5]. It turns out that the assumed presence of free and pinned vortices suffices also to explain the dependence of the damping coefficient Γ on the magnetic field.

The energy of the superconductor in an external magnetic field $\vec{H}_0$ is (see [5, 6])

$$\mathcal{E} = \int dV \left[\frac{N_s m v_s^2}{2} + \frac{(\mathbf{h} - \mathbf{H}_0)^2}{8\pi}\right], \tag{1}$$

where m is the mass and N_s the density of the superconducting electrons, $\vec{v}_s$ is the velocity of the superconducting condensate, and $\vec{h}$ is the magnetic field. To calculate the integrals in (1), it is convenient to put $\vec{h} = \vec{h}_0 + \vec{h}'$, where $\vec{h}_0 = \vec{H}_0 + \nabla\psi$ ($\nabla^2\psi = 0$, $\psi(\infty) = 0$, $(H_{0n} + \partial\psi/\partial n)_{r=R} = 0$, $\psi = R^2\vec{H}_0\cdot\vec{r}/r^2$) is the external field produced by the boundary field H_0 and failing to penetrate into the cylinder because of the Meissner effect; $\vec{h}'$ is the field produced by the vortices, viz., $-\vec{h}'_{in}$ inside the cylinder and

$$h'_{out} = \nabla\psi' \left(\Delta\psi' \leqslant 0,\ \psi'(\infty) = 0,\ \frac{\partial\psi'}{\partial n}\Big|_{r=R} = B_n,\ \psi' = -R^2\frac{\mathbf{B}\mathbf{r}}{r^2}\right)$$

outside the cylinder. Then, for example, the integral $(4\pi)^{-1}\int dV\vec{h}'(\vec{h}_0 - \vec{H}_0)$ is equal to

$$\int dV \frac{h'(h_0 - H_0)}{4\pi} = -V\frac{BH_0}{4\pi} + \int_{(out)} dV \frac{\mathrm{div}(h'\psi)}{4\pi} = -V\frac{BH_0}{2\pi}$$

(V is the volume of the cylinder). Here $\vec{B}$ is the magnetic induction inside the superconductor and is equal to the mean value of the field $\vec{h}'$: $\vec{B} = \vec{B}_p + \vec{B}_f$, $B_p = n_p\Phi_0$, $B_f = n_f\Phi_0$, $B_n = B_p \cos(\theta - \phi) + B_f \cos(\theta - \phi')$, where $\Phi_0 = \pi\hbar c/e$ is the magnetic-flux quantum, n_p and n_f are the densities of the pinned and free vortices. The remaining symbols are clear from Fig. 2, which shows the cross section of a cylinder of radius R; ϕ is the angle of rotation of the pinned vortices and coincides with the angle of cylinder rotation; ϕ' is the angle of

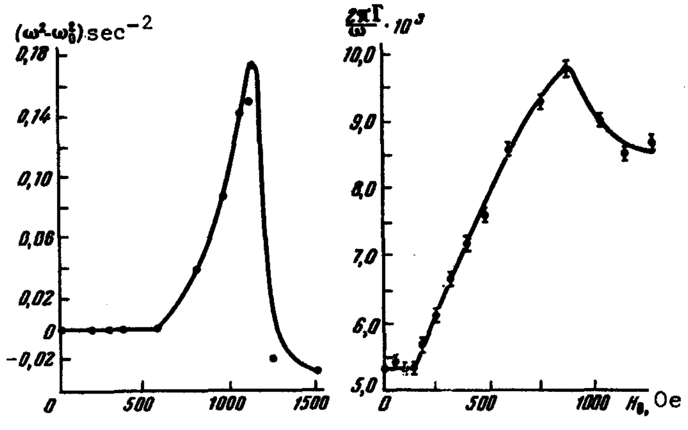

Fig. 1

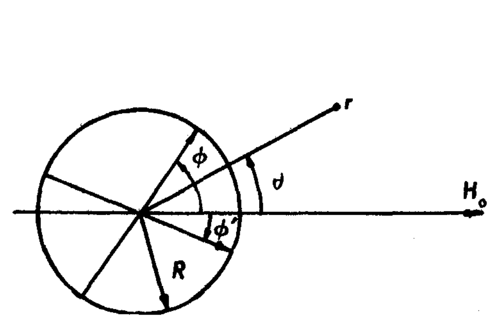

Fig. 2

rotation of the free vortices. It is assumed that the oscillation amplitude is small enough and the vortex filaments remain approximately straight.

Inside the superconductor, the quantities $\vec{h}'$ and $\vec{v}_s$ satisfy the Londons' equations

$$\mathrm{rot}\,\mathbf{h}'_{in} = \frac{4\pi}{c} N_s e \mathbf{v}_s \qquad \mathrm{rot}\,\mathbf{v}_s = -\frac{e}{mc}\mathbf{h}'_{in}$$

with the vortex filaments determined as singular solutions of these equations, subject to the quantization rule [5]

$$\oint_{C_r \to 0} \mathbf{v}_s d\mathbf{r} = \pi\hbar/m,$$

where C_r is an infinitesimally small contour surrounding the filament. Therefore, separating in natural fashion $\vec{h}'_{in} = \vec{h}_p + \vec{h}_f$ and $\vec{v}_s = \vec{v}_{sp} + \vec{v}_{sf}$, and integrating by parts, we get

$$\int_{(in)} dV \left[\frac{\mathbf{h}_f^2}{8\pi} + \frac{N_s m \mathbf{v}_{sf}^2}{2}\right] = \int_{(in)} dV \frac{mc}{8\pi e} \mathrm{div}(\mathbf{h}_f \times \mathbf{v}_{sf}) =$$
$$= \frac{\Phi_0}{8\pi} \sum_{i_f} \sum_{k_f} \int \mathbf{h}_f(\mathbf{r}_i - \mathbf{r}_k) d\vec{\ell}_i \approx V\left(\epsilon_0 n_f + \frac{B_f^2}{8\pi}\right),$$

where $\epsilon_0 = \Phi_0 h(0)/8\pi = \Phi_0 H_{c1}/4\pi$ is the logarithmically large ($\sim \ln\kappa$, $\kappa \gg 1$ is the parameter of the Ginzburg-Landau theory) is the self-energy of the vortex per unit length [5].

The remaining integrals on (1) are calculated in similar fashion, and the following potential, accurate to some insignificant constants, is obtained for the interaction of the cylinder and the external field

$$U = V\left[\epsilon_0 n_f + \frac{\Phi_0^2}{4\pi} n_f^2 + \frac{B_p B_f}{2\pi} - \frac{(B_p + B_f) H_0}{2\pi}\right]. \tag{2}$$

The motion of the free vortex filaments relative to the superconductor is accompanied by an energy dissipation described phenomenologically with the aid of the viscosity coefficient of the vortices [8]: $\eta = \Phi_0 \sigma_n H_{c2}/c^2$ (σ_n is the normal conductivity of the sample). It is easy to calculate the corresponding dissipative function Ψ and the oscillation damping coefficient Γ:

$$-\frac{1}{2}\dot{\mathcal{E}}_{mech} = \Psi = \int dV n_f \frac{\eta v_L^2}{2} = V n_f \frac{\eta R^2 (\dot\phi - \dot\phi')^2}{4} \qquad v_L = R(\dot\phi' - \dot\phi)$$
$$\Gamma = \Gamma_0 + \frac{\bar\Psi}{\mathcal{E}_{mech}} = \Gamma_0 + V n_f \frac{\eta R^2 \overline{(\dot\phi - \dot\phi')^2}}{4 I \overline{\dot\phi^2}} \tag{3}$$

v_L is the velocity of the vortex-filament element relative to the superconductor, I is the moment of inertia of the vibrating system, and Γ_0 is the damping due to other mechanisms.

Since the relaxation time of the free vortices is of microscopic order of magnitude and is small, the angle ϕ' in the potential U [Eq. (2)] is determined at each instant of time from the equilibrium condition

$$\partial U/\partial \phi' = 0. \tag{4}$$

As to the density n_f of the free vortices, it is apparently possible to have here either a slowing down of the relaxation, when the additional vortices do not have time to penetrate inside the sample during the oscillation, owing to the large surface barrier, or the opposite case of fast relaxation.

1. In the first case n_f = const. An elementary calculation with the aid of the equation of motion $I(\ddot{\phi} + \omega_0^2\phi) + (\partial U/\partial\phi) = 0$ and relations (2), (3), and (4) yields, in an approximation in which the amplitudes of ϕ and ϕ' are small,

$$\omega^2 = \omega_0^2 + \frac{VH_0 B(H_0)}{2\pi I}\,\frac{c_p\left(1 - \frac{B(H_0)}{H_0}\right)}{1 - c_p\frac{B(H_0)}{H_0}} \qquad B(H_0) = B_p + B_f\,,$$

$$\Gamma = \Gamma_0 + \frac{VR^2\sigma_n H_{c2} B(H_0)}{4Ic^2}\,\frac{c_f}{\left(1 - c_p\frac{B(H_0)}{H_0}\right)^2} \qquad c_p = \frac{B_p}{B(H_0)} \quad c_f = \frac{B_f}{B(H_0)}\,,$$

where ω_0 is the zeroth oscillation frequency and is connected with the elastic properties of the vibrating system; $B(H_0)$ is the equilibrium magnetic induction and can be determined from the Abrikosov magnetization curve [5][1].

2. In the opposite case of fast relaxation, the density n_f should also be determined from the equilibrium condition $\partial U/\partial n_f = 0$. Calculations similar to the preceding ones yield for fields that are not too weak, $H \gg H_{c1}$,

$$\omega^2 = \omega_0^2 + \frac{V\epsilon_0}{I}\,\frac{n_p}{1 - \frac{\Phi_0 n_p}{H_0}}\,, \tag{6}$$

$$\Gamma = \Gamma_0 + \frac{VR^2\sigma_n H_{c2}}{4Ic^2}\,\frac{1}{1 - \frac{\Phi_0 n_p}{H_0}}\,.$$

In formulas (5) and (6), $n_p \to 0$ as $H_0 \to H_{c2}$ $(B \to H_{c2})$, the sample goes over into the normal state, and the damping due to the vortices goes over into damping by the eddy currents, with $\Gamma_{ed} = VR^2\sigma_n H_{c2}^2/4IC^2$.

It is seen from these formulas that in the general case a peak due to the denominator $1 - B_p/H_0$ is superimposed on the monotonic growth of the damping coefficient $\Gamma \sim B(H_0)$ or $\Gamma \sim H_0$. In case (6) this peak is directly connected with the frequency peak:

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[1] When this curve is plotted for a cylinder in a transverse field, it is necessary to take the demagnetization factor into account.

$$\frac{\Gamma - \Gamma_0}{\Gamma_\Phi} = \frac{H_0}{H_{c2}} + \frac{l\Phi_0}{V\epsilon_0 H_{c2}}(\omega^2 - \omega_0^2).$$

From the physical point of view, the additional absorption is due to the repulsion of the free vortices from the pinned ones, which increases the oscillation amplitude of the free vortices relative to the superconductor and correspondingly increases the loss to viscous motion of the vortex filaments.

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INFLUENCE OF HIGH-FREQUENCY FIELD ON THE TUNNEL EFFECT

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Let a superconducting film of thickness $d \gg \zeta_0, \lambda$ (ζ_0 and λ are the coherence length and the field penetration depth, respectively) form a tunnel junction with some superconductor, and let an alternating magnetic field $H_0 \cos\omega$ be applied to the external surface of the film. At small ω the field does not penetrate into the junction under such conditions, and the film superconductivity parameter Δ is equal to its equilibrium value Δ_0 in the region of the junction. If $\omega = \Delta_0$, however, the situation may be somewhat different. It is shown in [1] that the superconductivity parameter of a bulky type-I superconductor is changed by the action of the alternating field of frequency Δ_0 up to distances on the order of the mean free path ℓ from the surface, in accordance with the formula

$$\frac{\Delta(r,t)}{\Delta_0} = 1 - \frac{e^2 H_0^2}{2c^2 q_0^4} \ln\frac{\ell}{r} \cos 2\Delta_0 t \equiv 1 - g(r)\cos 2\Delta_0 t \qquad (1)$$

with $\zeta_0 < r < \ell$; q_0 is the characteristic Pippard momentum [2]. It is assumed that in the absence of the field the superconductivity parameter has the same value near the surface as in the interior of the superconductor. Therefore, if the thickness $d \approx \ell \gg \zeta_0$ and $\omega = \Delta_0$, then the tunnel current is modulated at a frequency $2\Delta_0$, even though the alternating field does not penetrate directly into the junction. This is precisely the effect considered here.

As is well known, the tunnel current is expressed in terms of temporal Green's functions integrated with respect to the energy (cf., e.g., [3], formula (6)). The high-frequency electric field does not act directly on one of the superconductors of the junction, the equilibrium value of the gap is Δ_0', and the change of the Green's functions of the film in the region of the