density to decrease to one-half is approximately 4 msec. The time of free plasma expansion in a homogeneous magnetic field under conditions corresponding to Fig. 2 turned out to be of the order of 1.5 msec. The pulse-measurement data correspond qualitatively to the predictions of the theory (see [3, 4]).

The agreement between experiment and theory indicates that the method of longitudinal plasma containment with the aid of many probkotrons is promising, since the same theory predicts that, in the problem of expansion of a plasmoid with free boundaries, which is of interest for controlled thermonuclear fusion, the use of a multiple-mirror system increases the plasma lifetime by roughly L/ℓ times in comparison with the case of a homogeneous field.

Note added in proof. After this article was prepared for publication, we learned of a paper by Logan et al. (Phys. Rev. Lett. 29, 1435 (1972)), in which the theory was qualitatively confirmed. However, owing to the small number of mirrors (5), and the large transverse losses, a quantitative interpretation of the results given in that paper is difficult.

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CONCERNING ONE EXACT SOLUTION OF THE THEORY OF QUASILINEAR RELAXATION OF A PARAMETRICALLY UNSTABLE PLASMA IN THE FIELD OF POWERFUL RADIATION

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Research on parametric resonance in a plasma has stimulated a detailed development of individual aspects of the theory of such phenomena [1, 2]. Plasma heating in research on controlled thermonuclear fusion [3 - 5] or experiments with powerful radiation in the plasma near the earth [6] are but a few of the problems making the development of a theory for parametrically excited plasma pressing. In the present communication we report the first results of a quasilinear theory describing the establishment of a quasistationary state in a parametrically unstable plasma exposed to powerful radiation. The obtained self-similar solution of the system of quasilinear equations [7] makes it possible to determine the spectral density of the field fluctuations in the plasma, the electron distribution function, and the number of fast electrons arising during the course of the quasilinear relaxation in the development of parametric instability.

We consider a homogeneous plasma situated in a constant magnetic field[1)] and an alternating electric field (pump wave) with frequency ω_0 and intensity E_0. We confine ourselves to a study of quasilinear evolution of parametric instability, corresponding to the decay of the pump wave into an oscillation with a frequency of the lower hybrid resonance $\omega_{Le}|\cos\theta|$, and slow magnetosonic wave $\omega_{Li}kr_{De}|\cos\theta|$ (θ is the angle between the wave vector k and the magnetic field,

[1)] The results obtained here do not depend explicitly on the intensity of the external magnetic field, the role of which reduces in essence to making the quasilinear relaxation one-dimensional.

ω_{Le} and ω_{Li} are the Langmuir frequencies of the electrons and ions, and r_{De} is the electron Debye radius). Within the framework of the resonant interaction of the electrons with the excited high-frequency oscillations (the lower hybrid), the quasilinear relaxation is described by the equation[2]

$$\eta''(1 + e^{\eta}) + \eta'(\eta' - 1)\, e^{\eta} = 0 \qquad (1)$$

for the function $\eta(z)$ that determines the spectral energy density of the magnetic sound

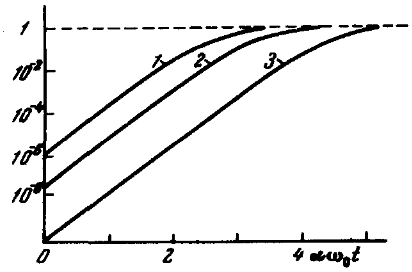
$$\frac{W_s(\mathbf{k}, t)}{N_e \kappa T_e} \qquad (2)$$

$$= \frac{16}{5}\left(\frac{2}{\pi}\right)^{1/2} \alpha C \, \frac{\omega_{L_e}^2}{\omega_0 \omega_{L_i}} \, \frac{e^{\eta(z)}}{k^5 r_{D_e}^2} \, \delta\left(|\cos\theta| - \frac{\omega_0}{\omega_{L_e}}\right).$$

Here N_e is the number of electrons per unit volume and $v_E = (\vec{e}\cdot\vec{E}_0/m\omega_0)$ is the rate of electron oscillations with charge e and mass m in the pump field. The self-similar variable z takes the form (v_{Te} is the thermal velocity of the electrons with temperature T_e and κ is Boltzmann's constant)

$$z \equiv -\ln\{k r_{D_e} \exp(\alpha\omega_0 t)\};$$

$$\alpha \equiv \left(\frac{2}{\pi}\right)^{1/2} \frac{C}{5} \, \frac{\omega_{L_e}}{\omega_{L_i}} \, \frac{v_E^2}{v_{T_e}^2}$$



Spectral energy density of slow magnetosonic wave (2) as a function of the time at different values of the wave number. The ordinates show, on a logarithmic scale, the quantity $1/5e^{\eta}$, which is equal to the ratio of the non-stationary spectral density $W_s(k, t)$ to the turbulent stationary density $W_s(k, \infty)$. The abscissas represent the dimensionless time

$$\alpha\omega_0 t = \left(\frac{2}{\pi}\right)^{1/2} \frac{C}{5} \, \frac{\omega_{L_e}}{\omega_{L_i}} \, \frac{v_E^2}{v_{T_e}^2} \, \omega_0 t .$$

The curves are plotted for three values of the wave number kr_{De}: 0.2 (curve 1), 0.1 (curve 2), and 0.03 (curve 3), at values of the self-similar constant

$$C = \frac{25}{4}\left(\frac{\pi}{2}\frac{\omega_0}{\omega_{L_e}}\right)^{1/2}\left(N_e r_{D_e}\right)^{-1/2} \frac{\omega_{L_i}}{\omega_{L_e}} \, \frac{v_{T_e}}{|v_E|} .$$

and characterizes the dependence of W_s on the wave number k and the time t. The general solution of (1) can be represented by the relation

$$C_2\, e^{\eta - z} = \left| 1 + \frac{e^{-\eta}}{1 + C_1} \right|^{C_1},$$

in which the integration constants C_1 and C_2 are determined accurate to the constant C which is a parameter of the self-similar problem:

[2] This equation arises under the conditions of interest to us as a consequence of the quasilinear system of two coupled equations for the distribution function of the electrons and of an acoustic noise (see [8]).

$$\left\{\frac{16}{5}\left(\frac{2}{\pi}\right)^{1/2} a\,C\,\frac{\omega_{L_e}^2}{\omega_0\,\omega_{L_i}}\,N_e r_{D_e}^3\right\}^{1/5} = k r_{D_e}\, e^{\eta + a\omega_0 t}\left(e^{-\eta} - \frac{1}{5}\right)^{6/5}. \quad (3)$$

The dependence of the noise energy (2) on the time is shown in the figure. It is seen from the figure that at large times, $(5/6)a\omega_0 t \gg 1$, the noise is saturated. The noise assumes a stationary value

$$\frac{W_s(\mathbf{k},\infty)}{N_e \kappa T_e} = \frac{32C^2}{5\pi}\,\frac{\omega_{L_e}}{\omega_0}\,\frac{\omega_{L_e}^2}{\omega_{L_i}^2}\,\frac{v_E^2}{v_{T_e}^2}\,\frac{1}{k^5 r_{D_e}^2}\,\delta\left(|\cos\theta| - \frac{\omega_0}{\omega_{L_e}}\right)$$

in accordance with the law

$$W_s(\mathbf{k},t) \approx W_s(\mathbf{k},\infty)\left\{1 - \left[16\left(\frac{2}{\pi}\right)^{1/2} a\,C\,\frac{\omega_{L_e}^2}{\omega_0\,\omega_{L_i}}\,N_e r_{D_e}^3\right]^{1/6} (k r_{D_e})^{-5/6} e^{-5/6 a\omega_0 t}\right\}.$$

The increment γ of the parametric buildup is obtained by differentiating (3) with respect to the time:

$$\gamma(\mathbf{k},t) = \frac{1}{2}\,\frac{\partial\eta}{\partial t} = \frac{a\omega_0}{2}\,\frac{5 - e^{\eta}}{1 + e^{\eta}}. \quad (4)$$

The exponential decrease of the increment with increase in time corresponds to the conclusion drawn above that the noise becomes saturated. To find the distribution function of the fast electrons with respect to the velocities $v = (\omega_0/k\cos\theta)$, it suffices to calculate the high-frequency damping decrement $\tilde{\gamma}$ of the lower hybrid with the aid of the increment (4) (γ_s is the decrement of the Cerenkov dissipation of the magnetic sound on the electrons)

$$\gamma = -\tilde{\gamma} + \frac{\omega_0}{16}\,\frac{\omega_s}{\gamma_s}\,\frac{v_E^2}{v_{T_e}^2}\,|\cos\theta|; \quad \gamma_s > \gamma,\ \tilde{\gamma}.$$

We verify with the aid of (3) and the explicit expression for $\tilde{\gamma}$

$$\tilde{\gamma}(\mathbf{k}\,t) = \frac{\omega_0}{\sqrt{2\pi}}\,\frac{\omega_{L_e}}{\omega_{L_i}}\,\frac{v_E^2}{v_{T_e}^2} - \frac{a\omega_0}{2}\,\frac{5 - e^{\eta}}{1 + e^{\eta}}$$

that the electron distribution function assumes in the region of the quasilinear relaxation of a parametrically unstable plasma a stationary value

$$F_e(v,\infty) = \frac{\sqrt{2}}{\pi^{3/2}}\,\frac{\omega_{L_e}}{\omega_{L_i}}\,\frac{v_E^2}{v_{T_e}^2}\,\frac{1}{|v|} + C_3. \quad (5)$$

The resultant integration constant C_3 is obtained from the condition for joining the distribution (5) with the Maxwellian distribution on the left and v_1 of the quasilinear diffusion velocity interval ($v_1 < v < v_2$)

$$C_3 = -\frac{\sqrt{2}}{\pi^{3/2}}\,\frac{\omega_{L_e}}{\omega_{L_i}}\,\frac{v_E^2}{v_2\,v_{T_e}}\left\{\ln\frac{v_2}{v_1} - \frac{v_{T_e}^2}{v_1^2}\right\};$$

$$v_1 \approx v_{T_e}\left\{2\ln\left(\frac{\pi}{2}\ \frac{\omega_{L_i}}{\omega_{L_e}}\ \frac{v_1 v_{T_e}}{v_E^2}\right)\right\}^{1/2}.$$

The fast-electron density δN_e corresponding to the distribution function (5) is given by the formula

$$\delta N_e = \frac{\sqrt{2}}{\pi^{3/2}} N_e \frac{\omega_{L_e}}{\omega_{L_i}} \frac{v_E^2}{v_{T_e}^2} \ln^{-1}\left(\frac{\pi}{2}\ \frac{\omega_{L_i}}{\omega_{L_e}}\ \frac{v_1 v_{T_e}}{v_E^2}\right). \quad (6)$$

In a hydrogen plasma $(\omega_{Le}/\omega_{Li}) \simeq 43$, subjected to the action of relatively weak radiation fluxes $(v_E^2/v_{Te}^2) \simeq 5 \times 10^{-4}$, the relative number of fast electrons $(\delta N_e/N_e)$ reaches, according to (6), a value on the order of several tenths of one per cent.

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LOW-TEMPERATURE NEGATIVE DIFFERENTIAL MICROWAVE CONDUCTIVITY IN SEMICONDUCTORS FOLLOWING ELASTIC SCATTERING OF ELECTRONS

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Generation of short-wave microwaves in sufficiently thick samples by using the volume negative differential conductivity (NDC) mechanisms, well known in microwave semiconductor electronics, encounters certain difficulties, particularly because NDC exists also at low frequencies. The volume NDC mechanism proposed in the present article leads to NDC only at sufficiently high frequency, and can hopefully be used to generate short-wave microwave radiation in samples having dimensions determined only by the cooling conditions.

Microwave NDC is possible in "pure" semiconductors at low temperatures in a constant electric field E such that the main mechanism of electron scattering is spontaneous emission of optical phonons by the electrons, namely, if[1)]

$$\nu p_0 \ll eE \ll p_0/\tau_0. \quad (1)$$

[1)] Galvanomagnetic effects under such conditions were considered in [1], and effects of "strong" alternating fields were considered in [2].