Conductivity of two-dimensional electrons in a strong magnetic field

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The conductivity of two-dimensional electrons in a strong magnetic field has been calculated in the low-density limit. It is assumed that electrons are scattered only by phonons.

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We examine the conductivity of a two-dimensional electron system in a strong magnetic field, which is produced by the scattering of electrons by phonons. This problem has been studied intensively recently in connection with other properties of systems of the metal-oxide-semiconductor type.[1-3] The magnetic field is assumed to be so strong that all the electrons are in the ground Landau level. We assume that the surface density n of electrons is so small that the condition $2\pi l_H^2 n \ll 1$ is satisfied. Here $l_H^2 = c\hbar/eH$ ($\frac{1}{2}\pi l_H^2$ is the maximum number of electron states in a given Landau level). This makes it possible to ignore the interaction of electrons with each other and to consider only their interaction with phonons. We place this system in an external electric field that is perpendicular to the magnetic field. We are interested in only that component of the current which is parallel to the electric field. We should consider the mechanism for electron drift in this direction. It is obvious that the collision of an electron with one phonon does not change the location of the center of the electron orbit. Because of the energy-conservation law, the electrons can interact only with phonons with a frequency $\omega_q = 0$ (a two-dimensional electron does not change its energy while residing in a given Landau level); this indicates that there is actually no interaction. A completely different situation occurs when two-phonon processes are taken into account. In fact, if first one phonon is emitted (or absorbed) and then a second phonon is absorbed (or emitted), the energy-conservation law requires (in the isotropic approximation) only that the moduli of the wave vectors of both phonons be equal, while their individual components may be different. In the gauge in which the vector potential of the magnetic field ($\mathbf{H} \parallel z$ axis and $\mathbf{E} \parallel x$ axis) is $A_x = A_z = 0$ and $A_y = Hx$, the wave function of the electron is $\psi(r) = Ae^{-(x-x_0)^2/2} e^{ip_y y}\phi(z), x_0 = cp_y/eH$, and the change in the momentum component p_y due to scattering by phonons represents a displacement of the electron along the x axis, which leads to the appearance of a current. Assuming that the interaction of electrons with phonons is weak, we confine ourselves to the two-phonon approximation. Since the magnetic length l_H is much greater than the atomic distances, we can confine ourselves to the second order in the linear electron-phonon interaction.

We calculate the conductivity by means of the kinetic equation obtained by using the method developed by Keldysh.[4] Instead of giving the rather cumbersome though standard intermediate calculations, we shall mention only their salient points. First, we ignore the corrections corresponding to the renormalization of the electron energy due to interaction with phonons; second, the ground state of noninteracting electrons corresponds to the energy of an individual electron (in the previously indicated gauge) $\epsilon_p = \epsilon_0 - el_H^2 E p_y$. Finally, the kinetic equation for the distribution function f_p of electrons with respect to the momentum $p \equiv p_y$, because the distribution function changes little at distances of the order of the magnetic length l_H, has the form of the Fokker-Planck equation ($\hbar = k = 1$)

$$\frac{\partial f_p}{\partial t} = \frac{\partial}{\partial p}\left[D\frac{\partial f_p}{\partial p} - \frac{el_H^2 E}{T} D f_p\right], \tag{1}$$

where the coefficient for the diffusion of electrons with respect to the orbit centers is

$$D = \frac{\pi}{2} g^4 \int (q_{1y} + q_{2y})^2 N_{q1}(1 + N_{q1})\delta(\omega_{q1} - \omega_{q2})\frac{d^3q_1 d^3q_2}{(2\pi)^6} \times e^{-q_1^2 l_H^2}\left[1 - e^{il_H^2(\mathbf{q}_{1\perp} \times \mathbf{q}_{2\perp})}\right], \tag{2}$$

J is the electron-phonon interaction constant, N_q is the phonon distribution function, and $(\mathbf{q}_{1\perp} \times \mathbf{q}_{2\perp}) = q_{1x}q_{2y} - q_{1y}q_{2x}$. We note that the phonons are assumed to be three-dimensional; in the calculation of the matrix element of the transition the dependence on the component q_z vanished because the characteristic phonon wavelengths are $\lambda_T \gg a$, where a is the localization dimension of an electron in the direction of the magnetic field. As a result, the dependence of this effect on the boundary conditions for the phonons is negligible.

It can be seen from Eq. (1) that the Einstein relation in this case is satisfied. The mobility in the coordinate space is (the conductivity is $\sigma = ne\mu$)

$$\mu = \frac{e l_H^4}{T} D . \tag{3}$$

After some manipulations, we obtain for the mobility μ ($\omega_q = sq$, $x = \omega_q/T$, and s is the velocity of sound)

$$\mu = \frac{e l_H^4 g^4}{32\pi^3 T s} \int_0^\infty \frac{e^x}{(e^x - 1)^2} q^6 e^{-q^2 l_H^2} A(q)\, dq, \tag{4}$$

where the function $A(q)$ is [$J_0(z)$ is the zero-order Bessel function]

$$A(q) = \iint_0^\pi \sin^3\theta_1 \sin\theta_2 [1 - J_0(q^2 l_H^2 \sin\theta_1 \sin\theta_2)]\, d\theta_1\, d\theta_2 . \tag{5}$$

The function $A(q)$ is

$$A(q) = \frac{8}{3} - \frac{2\pi}{z}\left[\mathbf{H}_0(z) - \mathbf{H}_1(z)\frac{1}{z}\right],$$

where $\mathbf{H}_{0,1}(z)$ is the Struve function, and $z = q^2 l_H^2$.

We examine two limiting cases. At low temperatures $T \ll T_0$ and $T_0 = s/l_H$ the calculations can be carried out, and we obtain

$$\mu = \frac{(2\pi)^7}{297 s^{12}} e l_H^8 g^4 T^{10}. \tag{6}$$

In the case of high temperatures $T \gg T_0$ the mobility is

$$\mu = \frac{e g^4 T}{32\pi^{5/2} l_H s^3}\left[1 - \int_0^{\pi/4} \frac{\sin^2\phi \sin(3\phi/2)}{\cos^{3/2}\phi \cos^{1/2} 2\phi}\, d\phi\right]. \tag{7}$$

We note that the corresponding formulas for the mobility for scattering by two-dimensional phonons are

$$\mu = \frac{e l_H^4 g^4}{8\pi s T} \int_0^\infty q^4 \frac{e^x}{(e^x - 1)^2} [1 - J_0(q^2 l_H^2)] e^{-q^2 l_H^2} dq . \tag{4'}$$

In the case of low temperatures,

$$\mu = \frac{2\pi^7}{15 s^{10}} e l_H^8 g^4 T^8, \tag{6'}$$

and in the case of high temperatures,

$$\mu = \frac{1}{8\pi s^3} e l_H g^4 T \int_0^\infty e^{-x^2}[1 - J_o(x^2) x^2 dx]. \tag{7'}$$

We now give some numerical estimates. We note that for $H \sim 10^5$ G the magnetic length is $l_H \sim 10^{-6}$ cm. Therefore, the temperature $T_0 \sim 0.1°$. We now assume that $g^2 = \beta^2 \epsilon_0^2/\rho s^2$, $\epsilon_0 = 1$ eV, ρ is the density of the semiconductor, β is a numerical parameter, and $s = \gamma 10^5$ cm/sec. Substituting these quantities in Eq. (7), we obtain for the mobility [α is the numerical coefficient in Eq. (7)]

$$\mu = \frac{a\beta^4}{32\pi^{5/2}\gamma^7} 10^5 T \text{ (abs. units)}. \tag{8}$$

We note that the uncertainty in the numerical value of the mobility, which is caused by the coefficients β and γ that have high powers, can reach several orders of magnitude. We obtain

$$\tau^* \sim \frac{a\beta^4}{\pi^{5/2}\gamma^7} 10^{-14} T \text{ sec.} \tag{9}$$

for the effective transit time $\tau^* = m\mu/e$. This value for τ^* is consistent with the data of Ref. 5, if the uncertainty of the numerical value of the coefficient is taken into account.

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