Absence of a vacuum condensate in supersymmetric gluodynamics

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(Submitted 7 September 1981)

Pis'ma Zh. Eksp. Teor. Fiz. **34**, No. 9, 508–510 (5 November 1981)

It has been shown without making use of perturbation theory that spontaneous supersymmetry breaking is missing in supersymmetric gluodynamics and that vacuum fermionic and gluonic condensates are not formed.

PACS numbers: 11.30.Qc, 11.30.Pb

The various field operators of quantum chromodynamics are known to possess nonvanishing vacuum expectation values in physical vacuum. Specifically, $\langle\bar{q}(x)\,q(x)\rangle$, $\langle G^{\alpha}_{\mu\nu}(x)\,G^{\alpha}_{\mu\nu}(x)\rangle \neq 0$, where q is the quark field, and $G^{\alpha}_{\mu\nu}$ is the stress tensor of the gluon field (a is the color index). The quark condensate $\langle\bar{q}q\rangle \neq 0$ indicates spontaneous violation of chiral invariance.[1] The gluon condensate $\langle G^2\rangle \neq 0$ in all probability is associated with color confinement. In any event, the connection between $\langle G^2\rangle > 0$ and the bag model can be traced.[2]

It is customarily thought that the condensate exists in every non-Abelian field theory with a strong coupling at large distances.

In this note we show that vacuum condensates are indeed missing in supersymmetric gluodynamics. It must be borne in mind, however, that supersymmetric gluodynamics[3] differs from conventional chromodynamics only in that the quark triplets are replaced by fermions in the adjoint representation.

A more detailed Lagrangian is written as follows:

$$\mathcal{L} = -\frac{1}{4}\, G^{a}_{\mu\nu} G^{a}_{\mu\nu} + \frac{i}{2}\, \bar{\lambda}^{a} \gamma_{\mu} D_{\mu} \lambda^{a}, \qquad (1)$$

where $D_\mu = \partial_\mu - igT^{\alpha}A^{\alpha}_{\mu}$ (g is the coupling constant), and T^{α} is the G-group generator; the vector field A^{α}_{μ} and the Majorana spinors λ^{α} are transformed according to the adjoint representation of the G group.

It can be shown[3] that, in addition to conventional gauge invariance, Lagrangian (1) has a new symmetry with respect to the transformation of bosons into fermions. The spinor charges Q_α corresponding to these transformations satisfy the conventional algebra

$$\{ Q_{\alpha}, \bar{Q}_{\beta} \} = -2(\gamma_{\mu})_{\alpha\beta} P_{\mu}. \qquad (2)$$

It is also rather simple to construct[3] the corresponding conserving currents $J_{\mu\alpha}$, so that $Q_\alpha = \int J_{0\alpha}\, d^3x$ (here P_μm is the total 4-momentum generator, and α and β are the spinor indices).

The existence of a quark or gluon condensate means that the supersymmetry is

 0021-3640/81/21485-03$ 00.60

broken spontaneously. This proposition follows from the relations

$$< \bar{\lambda}\lambda > = \frac{1}{4i} < \{ Q_{\alpha,} \; (\bar{\lambda}\gamma^{\mu}A_{\mu})^{a} \} > \quad [4] \qquad (3)$$

$$< T^{\mu}_{\mu}(x) > = < \{ Q_{\alpha}, (\gamma^{\mu} J_{\mu}(x))^{a} \} >. \qquad (4)$$

It should be taken into account that the path of the momentum energy tensor T^{μ}_{μ}, as usual, is proportional to G^2,

$$T^{\mu}_{\mu}(x) = \frac{\beta(g^2)}{2g^2} G^{a}_{\mu\nu}(x) G^{a}_{\mu\nu}(x), \qquad (5)$$

where $\beta(g^2)$ are Gell-Mann–Low functions. [In our case $\beta(g^2) = -3C_2(G)g^4/(16\pi^2) + O(g^6)$.] As a result, we see that nonvanishing $\langle\bar{\lambda}\lambda\rangle$ or $\langle G^2\rangle$ implies that $Q_\alpha|0\rangle \neq 0$. This means that the supersymmetry is broken spontaneously.

Further, it is clear that supersymmetry is not broken spontaneously in a finite-order perturbation theory, and that the condensates are missing. This is evident from the fact that spontaneous symmetry breaking is accompanied by the appearance of a massless particle, the so-called goldstino. The goldstino in this case can be only a bound state of λ and A. However, the bound states do not appear in finite-order perturbation theory (an excellent discussion of the general problems of supersymmetry, which are broached by us, may be found in a recently published preprint of Witten[5]).

Disregarding perturbation theory, we can assert, taking advantage of (5), that in this case the vacuum expectation values $\langle G^2\rangle \leqslant 0$. In fact, because of Lorentz invariance

$$< 0 | \, T^{\mu}_{\mu} \, | 0 > = 4E,$$

where E is the density of the vacuum energy. In supersymmetric theories, $E \geqslant 0$ always (see, e.g., Ref. 5), since the energy operator P_0 is determined non-negatively $P_0 \sim Q_\alpha{}^2$ [see Eq. (2)]. On the other hand, because of Lorentz invariance of the vacuum, we have for $\langle G^2\rangle < 0$

$$< 2\,(\mathrm{H}^{a})^2 > \equiv < \sum_{i,k} (G^{a}_{ik})^2 > = \frac{1}{2} < G^2 > < 0,$$

inconsistent with the non-negative specificity of the $(\mathrm{H}^{\alpha})^2$ operator. We obtain, therefore, a rigorous relation,

$$< G^{a}_{\mu\nu}(x) G^{a}_{\mu\nu}(x) > = 0. \qquad (6)$$

In other words, $E = 0$ and the supersymmetry is not broken spontaneously, $Q_\alpha|0\rangle = 0$. Finally, the absence of a quark condensate $\langle\bar{\lambda}\lambda\rangle = 0$ is now a direct consequence of the fact that $Q_\alpha|0\rangle = 0$ (see the discussion above).

Our proof is formal in nature, since the possible discrepancies of the considered values have not been discussed. It is very important to emphasize, therefore, that the quantity $\langle G^2\rangle$ can be analyzed only in the supersymmetric theory. In the other theories, in particular, in quantum chromodynamics, $\langle G^2\rangle$ has not been determined (it diverges) in the perturbation theory. It is therefore logical to introduce the average of the N-ordered product of the operators $\langle 0|:G^2:|0\rangle$. In this case all the features of the known signs are lost. In supersymmetric gluodynamics $\langle G^2\rangle = 0$ in the perturbation theory, and the N-product coincides with the total product for which the signs are known.[1)]

Supersymmetric gluodynamics, therefore, is an extremely rare example of a theory with a strong coupling constant in a four-dimensional space-time, in which the problem of spontaneous symmetry breaking can be solved exactly. Unfortunately, the method used to analyze it cannot be applied directly to other supersymmetric theories. The analyzed example, however, can adequately demonstrate the limitations of non-Abelian theories, in which the solution depends on the fermionic representation. In particular, we can assume that color confinement can be eliminated from supersymmetric gluodynamics. The vanishing of the fermionic condensate raises doubt about the fact that the condensate is always produced[6] when the coupling constant in the maximum-attraction channel is sufficiently large.

We wish to thank A. I. Vaĭnshteĭn and V. A. Novikov for in-depth discussions and critical comments.

[1)]In the zero-order coupling constant, $\langle G^2\rangle$ does not have fermionic loops, and it may seem perplexing that quantum chromodynamics differs from supersymmetric gluodynamics. The answer is that the zero-order coupling constant vanishes because of the equations of motion for $G_{\mu\nu}$. Both bosonic and fermionic loops appear in the next orders.

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Translated by S. J. Amoretty
Edited by Robert T. Beyer