Quasi-Alfvén waves, their self-excitation and spontaneous radiation

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New type of low-frequency waves, which we call thermodynamic quasi-Alfvén waves, can be self-excited in metals in which the electron and hole densities are the same, if a temperature gradient and a magnetic field parallel to it both exist. The frequency of these waves is proportional to the temperature gradient. Under conditions that can easily be achieved in the experiment, these waves are self-excited and a spontaneous rf radiation is emitted from the metal in which this occurs.

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The conditions under which electromagnetic waves can be self-excited in metals in a strong, external magnetic field in the presence of a temperature gradient were specified in Ref. 1. The rf emission of two types of self-excited waves was investigated in Ref. 2: helicoidal waves in the case of unequal electron and hole densities ($n_- \neq n_+$) and Alfvén waves in the case of equal densities in even-valence metals and, in particular, in bismuth. The case in which the waves propagated along the temperature gradient $\vec{\nabla} T$ and along the magnetic field $\boldsymbol{B}$ parallel or antiparallel to it (the z axis is directed along the temperature gradient $\vec{\nabla} T$) has also been investigated. Under real conditions the thermomagnetic Alfvén waves can appear in a metal such as bismuth, in which the following inequality is satisfied:

$$ku_A > \nu_{\mp},$$

where k is the wave vector of these waves, $u_A = B/\sqrt{4\pi n(m_- + m_+)}$ is their propagation velocity, and $\nu_{\mp}$ is the effective collision frequency of the carriers. This inequality cannot be satisfied in ordinary metals with a large carrier density. In the presence of $\vec{\nabla} T$, however, the self-excited waves, for which the inverse inequality

$$ku_A < \nu_{\mp} \tag{1}$$

is satisfied, can occur. Such waves appear in a strong magnetic field if the cyclotron frequency $\Omega = eB/(m_{\mp} c) \gg \nu_{\mp}$ (this is necessary in order that the Hall current vanish at $n_- = n_+$; c is the absolute charge). These waves, whose frequency is $\omega \ll \nu_{\mp}$, are called quasi-Alfvén thermomagnetic waves.

1. Condition for instability. In the presence of $\vec{\nabla} T$ and a constant magnetic field $\boldsymbol{B}$ the current density is

$$\mathbf{j} = \sigma \mathbf{E} + \eta [\mathbf{B}' \vec{\nabla} T] + \eta_1 [\mathbf{B}[\mathbf{B}' \vec{\nabla} T]], \tag{2}$$

where η and η_1 were determined in Ref. 2, $B' \propto E \propto \exp(ikz - i\omega t)$ are the fields of the waves, $k = k' + ik''$, and $\omega = \omega' + i\omega''$. Inserting (2) in Maxwell's equations, we obtain the dispersion relation

$$\omega = \frac{1}{2\sigma}\left(ck_z\eta\nabla T + i\mathscr{P}c\eta_1 k_z B_z\nabla T - i\frac{c^2k^2}{4\pi}\right). \qquad (3)$$

The quasi-Alfvén waves are circularly polarized, and the factor $\mathscr{P} = \pm 1$ for a right- and left-handed circularly polarized wave. For a real k the imaginary part of the frequency ω'' may be positive, indicating a buildup of the corresponding wave. If $\eta_1 B_z > 0$, then, if the temperature gradient is sufficiently large,

$$\nabla T > \nabla T_{cr} = \frac{ck}{4\pi|\eta_1|B} \qquad (4)$$

or if the magnetic field is not too strong,

$$\frac{m_{\mp}c\nu_{\mp}}{e} < B < B_{max} = \frac{4\pi|\eta_1|B^2\nabla T}{ck}, \qquad (5)$$

there is a buildup of the right-hand circularly polarized wave which propagates in the positive direction of the z axis, for which $k_z > 0$ (we call it a direct wave) and of circular polarized, backward wave, for which $k_z < 0$ (the subscript z denotes the vector component which may have a plus or a minus sign, in contrast to the k and B moduli). If $\eta_1 B_z < 0$, then the circularly polarized waves switch places. If the conditions (4) and (5) are satisfied, then the real frequency ω' and the wave vector k' satisfy the inequalities

$$\omega' < \omega'_{max} = \frac{2\pi|\eta_1 \mathrm{Re}\,\eta|B\nabla T^2}{\mathrm{Re}\,\sigma}$$
$$k' < k'_{max} = \frac{2\mathrm{Re}\,\sigma\,\omega'_{max}}{c|\mathrm{Re}\,\eta|\nabla T}. \qquad (6)$$

In this frequency region there is a convective instability of quasi-Alfvén waves in the unbounded medium. It is easy to see that an absolute instability occurs at the saddle point[3] $(\partial\omega/\partial k)_s = 0$ at the frequency

$$\omega'_s = \mathscr{P}\frac{\eta_1 \mathrm{Re}\,\eta\, B_z\nabla T^2}{\mathrm{Re}\,\sigma}$$

$$k_s = 2\pi(\mathscr{P}\eta_1 B_z\nabla T - i\eta\nabla T)$$

2. Self-excitation of quasi-Alfvén waves in an unbounded medium. Suppose that the sample under investigation is a plate of thickness d, whose plane is perpendicular to the z axis, and its dimensions in the x plane are large compared with d, so that the edge effects are negligible. If the forward wave increases in strength, we easily see that the wave reflected from the crystal surface also increases. Thus, there is a feedback, and the condition for the field equivalence[2,3] is given by

$$q_+ q_- \exp[i(k_+ - k_-)d] = 1, \tag{7}$$

where the plus and the minus signs pertain to the forward and backward waves. A reflection of a right-hand circularly polarized wave for which $k_+ \approx k'_{r+} + ik''_{r+}$ produces a left-hand circularly polarized backward wave for which $k_- = K'_{l-} + ik''_{l-}$. These two waves constitute one branch of self-exciting oscillations in the crystal. The waves $(l+, r-)$ comprise the second oscillation branch. Since the equivalency conditions for each branch must be satisfied independently, the q_+ and q_- coefficients in Eq. (7) must be written as q_{r+} and q_{l-}, denoting the reflection coefficients of the right-hand circularly polarized forward wave at the boundary $z = d$ and of the left-hand circularly polarized backward wave at the boundary $z = 0$

$$q_{r+} = -\frac{ck_{r+} - \omega}{ck_{l-} - \omega} = q^{\circ}_{r+} e^{i\phi_{r+}} \quad q_{l-} = -\frac{ck_{l-} + \omega}{ck_{r+} + \omega} = q^{\circ}_{l-} e^{i\phi_{l-}}. \tag{8}$$

The relation such as (7) also applies to waves of the second branch. To satisfy (7), we must have the equality

$$(k'_{r+} - k'_{l-})d + \phi_{r+} + \phi_{l-} = 2\pi p, \qquad p = 1, 2, 3, \dots \tag{9}$$

Since $c|k| \gg \omega'$ and $|k''| \ll k'$, the quantities $\phi \ll 1$ and hence the spectrum for the k' values change negligibly in the presence of radiation at the boundaries. Using (9), we write (7) in the form

$$k''_{l-} - k''_{r+} = -\frac{1}{d}\ln(q^{\circ}_{r+} q^{\circ}_{l-}) \approx -\frac{2}{d}\ln q^{\circ}. \tag{10}$$

According to Ref. 4, the instability region can be determined by inserting the expressions $k = k' + ik''$ and $\omega = \omega' + i\omega''$ in Eq. (3). Solving the obtained equation for k'' and inserting it in (10), we obtain the correlation

$$\omega'' = \frac{\eta_1 B_z \nabla T}{2\omega' \Gamma} \frac{c}{d} \ln q^{\circ} - \frac{\mathrm{Re}\,\sigma}{2\Gamma}, \qquad \Gamma = \sum_{-,+} \frac{nmc^2}{B^2}. \tag{11}$$

A buildup of waves of the relevant branch occurs in that frequency region ω' in which $\omega' > 0$. The values $k' = f(\omega', \omega'')$ in this case should differ slightly from the values $2\pi p/d$; the frequencies ω' are determined by these values of k'. The waves are attenuated as a result of reflection from the boundaries, since $q^0 < 1$ due to radiation. During the time $\tau = 2d/u$ (where u is the group velocity) during which the wave packet travels a path $2d$, the field attenuation after two reflections from the boundaries is

$$(1 - q^{\circ}_{r+})(1 - q^{\circ}_{l-}) \approx 1 - 2q^{\circ} = \frac{2\omega' + \phi c/d}{c|k'|} \ll 1.$$

It is easy to see that the average oscillation decrement

$$\gamma = \frac{u}{2d}(1 - 2q^{\circ})$$

is much smaller than the increment ω''. The flux density of the emitted energy is

$$S = \frac{c}{\pi}\left(\frac{\omega'}{c k'}\right)^2 (B')^2 \ . \qquad (12)$$

3. Steady radiation flux. The nonlinear processes limit the buildup of waves, which is completed at a certain steady-state amplitude. Since the method of its calculation was described in Ref. 2, we shall limit ourselves to reporting the results. The density of that part of the linear current with respect to the wave field which is proportional to the supercritical temperature gradient $\delta\nabla T$ is

$$j_{1x} = \delta \nabla T(\eta_1 B_z B_x' + B_y' \,\mathrm{Im}\eta^*) \cos (k' z - \omega' t) \ ,$$
$$j_{1y} = \delta \nabla T(\eta_1 B_z B_y' + B_x' \,\mathrm{Im}\eta^*) \sin(k' z - \omega' t) \qquad (13)$$

and the current density, which is cubic with respect to the variable fields, is comprised of that part which has a frequency ω and which is equal to $j_3(\omega)$, and a part which has a frequency 3ω and which is equal to $j_3(3\omega)$. The $j_3(\omega)$ current is important in our case. It was shown in Fig. 2 that the steady-state amplitude of the magnetic field B' corresponds to the equality

$$j_1(\omega, \delta \nabla T) + j_3(\omega, \nabla T_{cr}) = 0$$

from which

$$\frac{(B')^2}{B^2} \approx \delta \nabla T \left| \eta_1 B_z \right| \left\{ \sum_{-,+} \left(| \eta_1 B_z \nabla T_{cr} | + | \frac{\omega'}{c k_z'} \mathrm{Im}\sigma^* | \right) \right\}^{-1}; \quad \delta \nabla T << \nabla T_{cr}.$$

Inserting this expression in (12), we obtain

$$S = \frac{c}{\pi} \left(\frac{\omega'}{c k'}\right)^2 B^2 \delta \nabla T \left| \eta_1 B_z \right| \left\{ \sum_{-,+} \left(|\eta_1 B_z \nabla T_{cr}| + | \frac{\omega'}{c k_z'} \mathrm{Im}\sigma^* | \right) \right\}^{-1} . \qquad (14)$$

4. We have obtained the following estimates (for $T = 5$ K and $\nabla T / = 3$ deg/cm). Suppose that $d = 0.5$ cm, $B = 10^4$ Oe, $\sigma \approx 3 \times 10^{17}$, $\eta \approx 10^{25}$, and $\eta_1 \approx 3 \times 10^{23}$. For beryllium,[5] a compensated metal, the flux density of the radiated energy is $S \approx (1 - 10)$ W/cm^2 and $\omega \approx 10^4$ sec^{-1}.

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