Parametric bleaching of dense plasmas

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A mechanism is proposed for the nonlinear bleaching of a dense plasma slab. In this new mechanism, the electromagnetic wave incident on the plasma decays into plasma waves and then reappears as a result of the coalescence of the plasma waves at the second boundary of the slab.

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The transmission of electromagnetic signals through dense plasmas, which are opaque from the standpoint of linear electrodynamics, is a question of practical importance. The possible nonlinear penetration of electromagnetic waves through a dense bounded plasma has previously been linked to a density redistribution caused by ponderomotive forces[1–4] and to echo effects.[5,6]

In this letter we wish to propose a new mechanism for the nonlinear bleaching of plasma slabs. This new mechanism is based on the parametric decay of an incident electromagnetic wave into plasma waves, for which the plasma is transparent, followed by the regeneration of the electromagnetic wave at the second boundary of the slab as a result of the coalescence of the plasma waves. For definiteness, we assume a plasma slab which is strongly magnetized ($\Omega_e \gg \omega_{\mathrm{Le,max}}$) and which is inhomogeneous along the x axis. Incident normally on this slab is an ordinary electromagnetic wave whose frequency ω_0 lies between the electron and ion plasma frequencies. A magnetic field is imposed parallel to the slab boundary, along the z axis. A decay instability results in the excitation of oblique plasma waves with frequencies $\omega \sim \omega_0/2$, and these waves are trapped near the plasma density maximum. Describing the pump wave and the potential of the excited waves by

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{e}_z [E_0(x) \exp(i\omega_0 t) + E_0^*(x) \exp(-i\omega_0 t)],$$

$$\Phi(\mathbf{r}, t) = [\phi_0(x) + \phi_1(x) \exp(i\omega_0 t)] \exp(-i\omega t + ik_y y + ik_z z)$$

we can write the following system of equations to describe the interaction of these waves:

$$\frac{d^2\phi_0}{dx^2} - (k_y^2 + k_z^2 \epsilon(\omega, x))\phi_0 = i\frac{ek_y k_z}{m\omega^2 \omega_0 \Omega_e} E_0^2 \left(\frac{d}{dx} \frac{\omega_{Le}^2}{E_0}\right)\phi_1, \quad (1)$$

$$\frac{d^2\phi_1}{dx^2} - (k_y^2 + k_z^2 \epsilon(\omega - \omega_0, x))\phi_1 = -i\frac{ek_y k_z}{m(\omega - \omega_0)^2 \omega_0 \Omega_e} E_0^{*2} \left(\frac{d}{dx} \frac{\omega_{Le}^2}{E_0^*}\right)\phi_0, \quad (2)$$

$$\frac{d^2E_o}{dx^2} + \frac{\omega_o^2}{c^2}\epsilon(\omega_o, x) E_o = -i\frac{\omega_o}{c^2}\frac{e}{m}\frac{k_y k_z \phi_o \phi_1^*}{\omega(\omega - \omega_o)\Omega_e}\frac{d\omega_{Le}^2}{dx}, \quad (3)$$

where $\epsilon(\omega,x) = 1 - \frac{\omega_{Le}^2}{\omega^2}\left(1 - i\frac{\nu}{\omega}\right)$ is the linear dielectric constant of the plasma.

Assuming that the signal, which penetrates through the slab, is weak in comparison with the incident signal, we will take the following approach to solve the system of equations: For a given density profile we will ignore the right side of Eq. (3) and use this equation to find the distribution $E_0(x)$ corresponding to the linear theory [a linear approximation of the density profile at the slab boundary, for example, leads to an Airy function for $E_0(x)$]. The result found for $E_0(x)$ can be used to solve Eqs. (1) and (2) by perturbation theory, under the assumption that $\omega - \omega_n \equiv \delta$ and $\Delta = \omega_0 - 2\omega$ are small in comparison with ω_n, which is the eigenfrequency of the operator $\hat{L} \equiv \frac{d^2}{dx^2} - k_y^2 - k_z^2 \mathrm{Re}\,\epsilon(\omega,x)$ (see Refs. 7 and 8, for example). This approach leads to the following expression for the instability growth rate $\gamma = \mathrm{Re}\delta$:

$$\gamma = -\frac{\nu}{2} + \left\{ \frac{e^2 k_y^2 k_z^2}{m^2 \omega^2 (\omega - \omega_o)^2 \omega_o^2 \Omega_c^2} |\Lambda_n|^2 - \frac{\Delta^2}{4} \right\}^{1/2}, \quad (4)$$

where $|\Lambda_n|^2$ represents the overlap of the regions in which the interacting waves exist:

$$|\Lambda_n|^2 = \left| \int_{-\infty}^{\infty} dx\, \psi_n^2(x) E_o^2(x) \frac{d}{dx}\frac{\omega_{Le}^2}{E_o} \right|^2 \Big/ \left| \int_{-\infty}^{\infty} dx\, \psi_n^2(x) \right|^2. \quad (5)$$

Here $\psi_n(x)$ is the eigenfunction of the operator L which corresponds to the eigenvalue ω_n. For a parabolic density profile, the eigenfunction $\psi_n(x)$ can be expressed in terms of Hermite polynomials. From (4) we see that the instability threshold and the maximum growth rate correspond to $\Delta = 0$. Substituting the result for $\psi n(x)$ in the right side of (3), we can write a solution of this equation which describes a wave that has been transmitted through the slab (cf. Ref. 9):

$$E_o(x) = E_+(x)\int_{-\infty}^{x} \frac{dx'}{W} E_-(x') S(x') + E_-(x)\int_{x}^{\infty} \frac{dx'}{W} E_+(x') S(x'), \quad (6)$$

where

$$S(x) = -i\frac{e}{mc^2}\frac{\omega_o}{\omega_n^2}\frac{k_y k_z}{\Omega_e}\frac{d\omega_{Le}^2}{dx}\psi_n^2(x) e^{2\gamma t},$$
$$W = E_+' E_- - E_-' E_+, \quad (7)$$

and $E_\pm(x)$ are the solutions of the homogeneous version of Eq. (3), defined in such a manner that $E_+(x) \to 0$ in the limit $x \to -\infty$, while $E_-(x)$ corresponds to a wave propagating away from the slab. In the limit $x \to \infty$ we thus have

$$E_0(x \to \infty) = E_-(x) \int_{-\infty}^{\infty} dx' \frac{E_+(x') S(x')}{W} \tag{8}$$

The factor in (7), which increases over time, reflects the onset of an absolute parametric instability. Because of this factor, we must incorporate some mechanism for stopping the instability if we wish to determine the steady-state levels of the noise and of the outgoing signal. Skipping the details, we assume that the steady-state noise amplitude is comparable to the amplitude of the pump wave; this assumption agrees with an estimate from the cascade saturation theory,[10] for example. Taking this approach, we can estimate the maximum value of $k_y k_z e^{2\gamma t}\psi_n^2$ in (7) to be $E_{0,\text{inc}}^2$. For the energy flux propagating away from the slab, q, we then find the following estimate:

$$q = v_{gr} \frac{|E_0(\infty)|^2}{4\pi} \approx v_{gr} \frac{|E_-|^2}{4\pi} \frac{E_0^4}{W^2} \frac{16e^2}{m^2 c^4 \omega_0^2 \Omega_e^2} \left| \int_{-\infty}^{\infty} E_+(x')\psi_n^2(x') \frac{d\omega_{Le}^2}{dx'} dx' \right|^2, \tag{9}$$

where $v_{gr} = \partial\omega_0/\partial k_0 \approx c$ is the group velocity of the outgoing wave.

As an example we consider a slab of a homogeneous plasma with sharp boundaries at $x = \pm a$. The eigenfunction $\psi_n(x)$ is then described by sinusoidal functions, and the solutions $E_\pm(x)$ are

$$E_-(x) = \exp[\eta(x-a)], \quad \eta^2 = \frac{\omega_{Le}^2 - \omega_0^2}{c^2},$$

$$E_+(x) = \exp[-\eta(x-a)] + \frac{(\eta + ik_0)^2}{\eta^2 + k_0^2} \exp[\eta(x-a)], \quad k_0^2 = \frac{\omega_0^2}{c^2}. \tag{10}$$

The energy flux density of the radiation transmitted through the slab is thus found from (9) to be

$$q \approx 8 q_0 \frac{e^2 E_0^2}{m^2 \omega_0^2 c^2} \frac{\omega_{Le}^4}{(\omega_{Le}^2 - \omega_0^2)\Omega_e^2}, \tag{11}$$

where $q_0 = cE_0^2/4\pi$ is the energy flux of the radiation incident on the slab. We see that q is independent of the thickness of the slab, and it increases as ω_0 approaches ω_{Le} [it should be kept in mind that expression (11) is written under the assumption $\omega_0 \neq \omega_{\text{Le}}$]. Since the transmission of the slab falls off exponentially with increasing slab thickness in linear electrodynamics, the energy flux transmitted through a thick plasma slab is determined by the nonlinear mechanism described here. Other bleaching mechanisms may be less important in a highly magnetized plasma, because the ponderomotive forces fall off with increasing Ω_e and also because the effect of these forces depends on the slab thickness.

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