Production of gauge bosons in a conformally plane space

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The density of gauge bosons produced in a conformally plane space is calculated. The dependence of renormalized charges on the coordinates is discussed.

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Our universe is described by the Friedmann model to a high degree of accuracy. Studies which analyze the cosmological isotropic models with allowance for the one-loop, quantum-gravitational effects that have experimentally verifiable consequences have recently been published.[1,2] It would therefore be desirable to study in greater

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detail the quantum fields in a conformally plane space. We shall analyze the region of greatest interest $|R_{iklm}R^{iklm}| \gg m^4$, in which the rest mass m of particles can be ignored in first approximation. We shall assume that the fields obey the conformally invariant equations as $m \to 0$. It has been thought for some time that massless particles cannot be produced in a conformally plane world. If, however, the particles interact with each other, then the field equations can be considered effectively conformally noninvariant due to radiative corrections, and particle production is possible.[3–5]

We shall analyze the interaction of quantum spinor, scalar, and massless vector fields in an isotropic space with a metric

$$g_{ik} = a^2(x)\,\eta_{ik}\,, \tag{1}$$

where η_{ik} is the metric of the Minkowski space. Below we shall assume that the metric (1) does not have singularities and analyze the space-time in a classical manner.

We shall transform conformally the metric (1) $g_{ik} \to a^{-2} g_{ik}$, and the spinor and scalar fields $\psi \to a^{3/2}\psi, \phi \to a\phi$.

The renormalized field Lagrangian therefore is given by

$$L = -\frac{1}{4g_R^2(x)} \sum_i (F^i_{\mu\nu}(x))^2 - j_\mu(x) A^\mu(x) + \dots, \tag{2}$$

where the dots represent the terms containing the spinor and scalar fields but not A_μ, j_μ is the current of these fields, A_μ is the renormalized vector field, and $g^2 R^{(x)} \sim g^2 - \beta(g^2)\ln[a^2(x)/a^2(x_0)]$, where $\beta(g^2)$ is the standard Gell-Mann–Low function in Minkowski space. We have replaced the normalization momentum $\mu \to \mu a(x)$, which follows from Refs. 3 and 5. Hence we obtain $\partial g_R^2/\partial \ln a^2 = -\beta(g_R^2)$. We ignore the nonlocal terms in (2) and assume that the condition $|R_{iklm}R^{iklm}| \gg m^4$ is satisfied.

We shall represent the Lagrangian (2) in the form $L = L_0 + L_{\rm int}$, where

$$L_{int} = \frac{1}{2}\kappa F^i_{\mu\nu} F^{i\mu\nu} + \dots, \tag{3}$$

$\kappa(x) = [\beta(g^2)/g^2]\ln[a(x)/x_0)]$ is the effective coupling constant. We assume that

$$|\kappa| \ll 1. \tag{4}$$

The relation (3) is written in the first order in κ.

Performing calculations based on the perturbation theory, we obtain for the density of the produced gauge bosons

$$n = \frac{\beta^2(g^2)K}{8\pi g^4} \int \frac{d^4q}{(2\pi)^4}\,\Theta(q^2)\,(q^2)^2\,|\sigma(q)|^2, \tag{5}$$

where $\sigma(x) = \ln[a(x)/a(x_0)]$, $\sigma(q) = \int \sigma(x) e^{iqx} d^4x$, and k is the number of particle species; for SU(N) $K = N^2 - 1$.

The multiplier $\Theta(g^2)$ in (5) generally prevents transformation of this expression

into an integral in d^4x. Nonetheless, if the condition $q^2 \geqslant 0$ is satisfied for all $\sigma(q) \neq 0$, then n can be represented as $n = \int W(x^i) d^4x$, where

$$W(x^i) = \frac{\beta^2 (g^2) K}{8 \pi g^4} (\Box \sigma)^2. \tag{6}$$

Here $W(x^i)$ is the local probability for the production of a particle. $W(x^i)$ in (6) is a generalization of the corresponding value in Ref. 5, where it was determined for the special case of a weak field and for the value of σ which depends solely on time.

Adjusting Lagrangian (2) and ignoring the currents of the scalar and spinor fields, we obtain

$$D^\nu \left(\frac{1}{g_R^2(x)} F^i_{\mu\nu}(x) \right) = 0, \tag{7}$$

or in the linear (in $F^i_{\mu\nu}$) approximation,

$$(1 + 2\kappa) \partial^\nu F^i_{\mu\nu} + 2 \partial^\nu \kappa F^i_{\mu\nu} = 0. \tag{8}$$

Equation (8) generalizes the equation for propagation of the gauge boson, which was derived by Dolgov and Spokoĭnyĭ[5] for the case of the non-Abelian gauge group.

We must mention an important corollary of Eq. (2). The renormalized charge is a function of the physical momentum μ. In the conformal transformation of the metric (1) which was analyzed above, the physical momentum μ is converted to the momentum $\mu a(x^i)$ in the planar space-time, which depends on the coordinates. This leads to an effective dependence of the renormalized charge on the x^i coordinates, a fact which may have curious consequences. Linde and Weinberg showed (see, for example, Ref. 6) that there is spontaneous symmetry breaking in the Higgs model if $\lambda > 3e^4/32\pi^2$ (in the symbols of Ref. 6), and if $\lambda < 3e^4 32\pi^2$ the quantum corrections lead to a dynamic restoration of the symmetry. If $3e^4/32\pi\lambda \approx 1$, then, because of the indicated time dependence of the charges, a phase transition may occur at a certain moment of time in the cosmological problems. A thorough study of this phenomenon would be in order.

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