Theories with nonlocal electric charge conservation

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Theories in which the electromagnetic field strength is determined within a sign are analyzed. In such theories there may be filaments whose circumvention results in a change in the sign of the field strength. If filaments are present, there is no clear distinction between positive and negative charge.

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In this paper we will discuss a model for gauge fields which interact with scalar fields and fermions. We assume the gauge group G to be a connected, simple, compact, noncommutative group. Without any loss of generality, we may assume that the group G is singly connected. We denote the set of classical vacuums by R, while $H(\Phi)$ is the subgroup consisting of transformations which leave the classical vacuum Φ in place. [We assume that the Lagrangian contains a term $-V(\Phi) = -V(\Phi^1,...,\Phi^n)$, where $\Phi = (\Phi^1,...,\Phi^n)$ is an n-component scalar field, and the classical vacuum is defined as that point in the n-dimensional space at which $V(\Phi)$ reaches its maximum.] We will assume that the only degeneracy of the vacuum results from the effect of the group G (the group G acts on R in a transitive manner); then all the groups $H(\Phi)$ are isomorphic with the fixed group $H = H(\Phi_0)$, which corresponds physically to the group of symmetries that remain unbroken. All the existing "grand unified theories" are included in the class of theories described above. We will make the further assumption that the connected component of the group H is of the form $K \times U(1)$, where the group K is isomorphic (or locally so) with the product of simple noncommutative groups. The group $U(1)$ is then coupled with the electromagnetic field. In the theories with which we are concerned here the electromagnetic charge-conjugation operator is included in the gauge group G. In other words, we are assuming that the group H is not connected and that it contains the discrete symmetry α which "inverts" the group $U(1) \in H$ [i.e., it satisfies the condition $\alpha u \alpha^{-1} = u^{-1}$ for $u \in U(1)$]. A realistic example of this theory is the SO(10) grand unified model. A simpler example is the theory with the SO(3) gauge group and the scalar fields Φ^{ij}, which transform under a tensor representation of the group SO(3) and which form a symmetric tensor with a zero trace. We choose the potential $V(\Phi^{ij})$ in such a manner that the classical vacuum is a matrix with one double eigenvalue and one simple eigenvalue. The group H is then isomorphic with $O(2)$. As the transformation α, which "inverts" $U(1) = \text{SO}(2)$, we may choose a diagonal matrix for which $\alpha_{11} = \alpha_{33} = -1$, $\alpha_{22} = 1$ (we are assuming that the classical vacuum is described by a diagonal matrix for which the first two diagonal elements are equal). Fermions are inconsequential for the questions of interest here.

The theories of this class have some remarkable properties. First, the electromagnetic field tensor $F_{\mu\nu}$ is generally defined only within a sign, as follows from the expression derived for $F_{\mu\nu}$ in Ref. 1. We will not reproduce this entire expression, but we do wish to point out that in the region in which the scalar field is nearly at its vacuum value we have, $F_{\mu\nu}(x) = \langle G_{\mu\nu}(x), h\,[\Phi(x)]\rangle$, where $h(\Phi)$ is the generator of the electromagnetic subgroup $U(1) \in H(\Phi)$, $G_{\mu\nu}$ is the Yang–Mills field tensor, and $\langle\ ,\ \rangle$ denotes the invariant scalar product in the Lie algebra of the gauge group G. For the case in which we are interested, the normalization condition fixes the generator $h(\Phi)$ only within a sign; it is not possible to choose a branch of the double-valued function $h(\Phi)$ which is everywhere continuous and single-valued. The nonremovable double-valuedness of the function $h(\Phi)$ does not, on the other hand, mean that it is also impossible to distinguish a continuous and single-valued branch from the function $h\,[\Phi(x)]$. On the contrary, if the set of points x for which the electromagnetic field is defined is singly connected, such a branch can always be distinguished. Using the results of Ref. 2 we can construct fields which have axial symmetry and which do not change upon a displacement along the axis and which are such that the double-valuedness of the function $h\,[\Phi(x)]$ is nonremovable; for such fields, $F_{\mu\nu} \equiv 0$. These fields may be called "topologically nontrivial filaments"; the existence of filaments which have a finite linear energy density and which satisfy equations of motion was proved in Ref. 2. Using the "rectilinear" filaments which have just been described, we can construct closed filaments which satisfy the equations of motion only approximately. The fields of interest for physics have a finite energy. They may be thought of as consisting of "particles" (spheres in which the energy density is significantly nonzero) and closed filaments. Outside these particles and the filaments we assume that the field $\Phi(x)$ is appproximately the vacuum field; by this we mean, in particular, that the tensor $F_{\mu\nu}(x)$ is meaningful there. If there are no filaments, then $F_{\mu\nu}(x)$ may be assumed to be a single-valued function (if several nonintersecting spheres are removed from the space, we end up with a singly connected set; the situation is close to the standard situation). Topologically nontrivial filaments should arise, however, in the early stage of the evolution of the universe (at a phase transition). Furthermore, we cannot rule out the possibility that filaments which arose at that time still exist. The existence of filaments leads to several paradoxical conclusions. First, positive charge is indistinguishable from negative charge, and a magnetic monopole is indistinguishable from a magnetic antimonopole. If, however, two electric or magnetic charges are connected by a curve, we can determine the relative sign of these charges; it is determined by the field flux across the boundary of the region which contains the charges and across the curve connecting the charges. (This flux is determined within a sign, since the boundary of the region under consideration here is singly connected. If this flux is equal in modulus to the sum of the fluxes across the boundaries of the regions each containing one charge, then we must assume that the charges have the same sign; if this flux is instead equal to the difference, then the signs of the charges should be assumed to be different.) If there are no filaments, the relative sign of the charges does not depend on the choice of curve, so that once we have determined the sign of one of the charges we can determine the sign of all of them. If filaments do exist, and if the relative sign of the charges depends on the choice of curve, the nature of the interaction of the charges must depend on the particular curve the charges trace out as they close on each other. If there are no filaments within some sphere (in the solar system, for example), we can choose the sign of the charges within the system in a noncontradictory manner by connecting these charges by straight lines (or by any curves which lie within the sphere). If, however, an electron which is emitted from the sphere flies around a filament and returns to the sphere, it will be perceived as a positron and may annihilate with electrons that remained within the sphere. This phenomenon may be interpreted as a violation of the conservation of electric charge. Nevertheless, it may be assumed that charge conservation does hold if a definite charge is assigned to each filament. Let us cover each closed filament with a film, and let us assume for simplicity that these films do not intersect. If we remove the films and the "particles," we find a singly connected set. In it we can choose the tensor $F_{\mu\nu}$ unambiguously and continuously (at a film, the tensor $F_{\mu\nu}$ is discontinuous; as the film is approached from the different sides, the values of this tensor differ in sign). The electric and magnetic charges of the filament can be determined by examining the electric (or magnetic) field flux across a boundary in the vicinity of the film; this charge of course depends on the choice of film. It would be natural to place on a filament that film which it sweeps over during collapse; then the charge of the filament is equal to the net charge of the particles that are formed from the filament. For the definition above, we have conservation of electric and magnetic charges; this conservation follows from the circumstance that the tensor $F_{\mu\nu}$ satisfies Maxwell's equations. We should not be lulled into believing that the definition of the filament charge given above is completely free of paradoxes. If, for example, at some time in the future a filament will pass between the earth and the sun, then charge conservation requires that at present we must assume that the electrons of the sun are positrons. This terminology will be justified in an obvious manner after the filament has passed. For it to also be reasonable at present, we must assume that the charges that move from the sun to the earth change sign when they intersect the surface along which the filament will pass (whenever this happens!). The charge of the filament changes when the charges cross this surface. Consequently, charge conservation has a rather formal meaning in the situation under discussion here; all that we can say is that there is a nonlocal charge conservation, since the charge of a filament is determined by not only the points of the filament itself but also by the film.

In principle, these filaments could be observed. Let us assume that a filament passes through a region filled with ordinary matter (we will calculate the relative charge of the particles by connecting them with straight lines). After the filament has passed, ordinary matter remains on one side of the surface along which the filament passed, while antimatter is found on the other side. The emission of γ rays upon the subsequent annihilation could furnish evidence of the passage of a filament.

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