Production of fast electrons in Z pinches by an anomalous skin effect

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A mechanism is discussed for the production of fast electron beams in Z pinches at the time at which the plasma column is constricted.

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The acceleration of particles in Z pinches was discovered early in fusion research and has remained the object of intense study. Semenov's detailed review[1] describes the basic experiments, the parameters of the accelerated-particle beams, and several theoretical models. The electron beams which are produced exhibit the following properties: 1) The electron velocity approaches the speed of light, $v \approx c$, and the electron energy is $\epsilon \sim 0.3$–1 MeV; 2) the time at which the fast particles appear is approximately the time at which the constriction forms, and a hot plasma appears at the constriction; 3) the electron beam current satisfies $J_{\rm EB} \approx J$, where J is the current at the time at which the constriction forms; 4) the electron beam may have a tubular structure, as shown by Gribkov *et al.*[2] (see also Refs. 3 and 4); 5) the energy of the electron beams for plasma-focus devices is estimated to be $W_{\rm EB} \sim 10^2$–10^4 J; 6) the intensity of the hard x-ray emission, which is bremsstrahlumg emitted by the electron beam as it is stopped at various parts of the apparatus or in collisions with plasma ions, falls off in a power-law fashion at high energies; 7) the hard x-ray emission immediately precedes a soft x-ray emission by the hot plasma, according to several studies. Some mechanisms for the acceleration of heavy particles have recently been proposed (see Ref. 5, for example), but the appearance of accelerated electrons has yet to be explained.

In the final stage of the formation of the constriction in a Z pinch, the current drops off because of the sharp increase in the resistance at the constriction (because of the excitation of ion acoustic waves, for example). It is important to note that, although the resistance may increase by several orders of magnitude in a short time interval, the current decrease usually amounts to only a small fraction ($\lesssim 0.1$–0.2) of the total current, because of the comparatively large inductance of the discharge circuit. This change in the electrical parameters should lead in turn to a sharp increase in the electric field and to a skin effect at the constriction. If the resistance increases sufficiently rapidly, the electric field (E) may reach a level sufficient to satisfy the condition for particle runaway [$E > 1.6 \times 10^{-26}\,(n/T)$; Ref. 6].

Let us examine Maxwell's equations under the assumption that the runaway electrons carry the current in the constriction during the initial stage of the current decrease and that the change in the plasma density (n) over this time interval is small. As will be shown below, the skin thickness is much smaller than the transverse radius (r) of the plasma, so that we may treat the one-dimensional problem [$E(x)$, $n(x)$, $n(0) = 0$, where x is the transverse coordinate in the constriction; see Fig. 1], using $2\pi r$ as the length of the surface layer where necessary to obtain specific results. Transforming

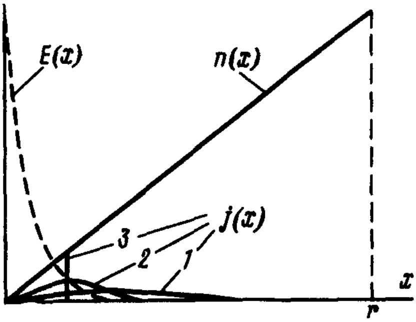

FIG. 1. Dynamics of the current density profile $j(x)$ during the electron acceleration. Curves 1, 2, and 3 correspond to successive stages in the acceleration.

Maxwell's equations, we find an equation for the electric field which describes the skin effect. In the present case (in which electron collisions are ignored), however, the coefficient in this equation turns out to be slightly different from that for the ordinary skin effect[6]:

$$\frac{d^2E}{dx^2} - \frac{4\pi e^2 n(x)}{mc^2} E = 0.$$

Now assuming that the plasma density at the edges of the plasma configuration increases in accordance with a power law, $n(x) \propto x\alpha$, we find

$$\frac{d^2E}{dx^2} - b\, x^{a} E = 0, \quad \text{where } b = \text{const}, \quad a > 0.$$

A solution of this equation, which vanishes in the limit $x \to \infty$, is[7]

$$E(x) = E_{max} A \sqrt{x} K_{1/(a+2)}\left(\frac{2\sqrt{b}}{a+2} x^{(a+2)/2}\right), \text{ where } E(0) = E_{max},$$

where $A = \frac{2}{\pi} \sin\left(\frac{\pi}{a+2}\right) \Gamma\left(\frac{a+1}{a+2}\right)\left(\frac{\sqrt{b}}{a+2}\right)^{1/(a+2)}$, and $K_\nu(x)$ is the modified Hankel function. For small values of x we easily find $E(x) = E_{max} - B(a)x$; the asymptotic behavior is $E(x) \propto x^{-a/4} \exp\left(-\frac{2\sqrt{b}}{a+2} a^{(a+2)/2}\right)$. For further calculations we will consider two cases: a) of the plasma focus (PF; $r \sim 10^{-1}$ cm, $n_0 \sim 10^{19}$ cm^{-3}, $J \sim 5 \times 10^5$ A, $\tau \sim 10^{-7}$ s, where τ is the plasma lifetime); and b) the plasma points (PP: $r \sim 10^{-3}$, $n_0 \sim 10^{21}$ cm^{-3}, $J \sim 10^5$ A, $\tau \sim 10^{-10}$ s). For $n(x) = n_0 x/r$, the characteristic depth to which the field penetrates into the plasma is

$$\Delta x \approx \left(\frac{l^{2} r}{4\pi n_0 e^2}\right)^{1/3} \begin{cases} \to 1.5 \cdot 10^{-3} \text{ cm for the } PF, \\ \to 0.7 \cdot 10^{-4} \text{ cm for the } PP. \end{cases}$$

In other words, the current flows along a thin surface layer, which contains a relatively few particles. The electric field near the plasma surface must therefore reach a high value in order to accelerate the electrons in the skin to the high velocity which these electrons must have in order to transport the current in the circuit across the constriction. This mechanism can operate if the characteristic time for the increase in the resistance at the constriction is shorter than the characteristic time required for the field to penetrate a depth Δx into a plasma with the parameters corresponding to the stage preceding the onset of the skin effect:

$$\Delta\tau \sim \frac{4\pi\sigma\Delta x^2}{c^2} \begin{cases} \nearrow 10^{-9} \text{ s for the } PF, \\ \searrow 10^{-11} \text{ s for the } PP, \end{cases}$$

where σ is the conductivity.This condition can apparently be met during the turbulent increase of the plasma resistance. At a qualitative level it can be seen that during the acceleration in the resultant field the current density $j(x)$ will vary in accordance with the situation shown in Fig. 1 [for the case in which the change in $n(x)$ is small during the acceleration and in which plasma quasineutrality is taken into account], and in the limiting case all the current-carrying electrons can reach a velocity $v \approx c$ and be concentrated in a surface layer of thickness $\Delta = \sqrt{J/\pi n_0 ec}$, where $n(x) = n_0(x/r)$. An estimate yields $\Delta \approx \Delta x$, and the situation is apparently close to this limiting case. The magnetic field, which reaches its maximum at the periphery of the plasma column, does not prevent the electron acceleration, since the Lorentz force in this case pushes the accelerated electrons into the plasma to some extent and is balanced by the force associated with the polarization electric field which results from this pushing. As a result, the forces acting along the radius of the plasma column are balanced, and the acceleration occurs along the axis of the column without obstruction. Assuming that at the end of the skin-effect zone the current flows through a surface layer of thickness Δx [the number of particles per unit length in the layer is $N = \pi n_0 \Delta x^2$ with $n(x) = n_0(x/r)$], we estimate the final electron velocity to be

$$v \approx \frac{J}{\pi n_0 \Delta x^2 e} \approx c,$$

in agreement with experiment. Adopting $\bar{\epsilon} \sim 10^5$ eV for the average electron energy, and assuming that the skin-effect stage lasts $\Delta t \sim 10.1\tau$, we estimate the total energy of the electron beam to be

$$W_{EB} \sim \frac{J\Delta t}{e}\bar{\epsilon} \begin{cases} \nearrow 10^3 \text{ J for the } PF, \\ \searrow 10^{-1} \text{ J for the } PP. \end{cases}$$

The hard x-ray emission caused by the accelerated electrons should evidently be detected at the leading edge of the pulse of soft x-ray emission, since the skin-effect stage precedes the formation of the hot plasma. Working in a similar fashion we can estimate the number of ions which are accelerated in the skin:

$$N_i \sim \frac{J\Delta t}{e}\sqrt{\frac{m}{M}} \begin{cases} \nearrow 10^{15} - \text{ for the } PF, \\ \searrow 10^{10} - \text{ for the } PP. \end{cases}$$

We can apparently assume that the profile of the electron energy along the transverse coordinate is approximately the same as $E(x)$, i.e., $\epsilon(0) = \epsilon_{max}$, followed by a monotonic decay. For small values of x we can then assume $\epsilon(x) = e_{max} - kx^{\beta}$ $(k,\beta > 0)$, and using the dependence $n(x) \propto x^{\alpha}$ we find $f(\epsilon) \propto (\epsilon_{max} - \epsilon) \frac{\alpha + 1 - \beta}{\beta}$ for the electron distribution function at high energies. This result can explain the power-law decay of the intensity of the hard x-ray emission with increasing x-ray energy.

In summary, the electron beams which appear in Z pinches can be explained by the mechanism outlined in this letter, which incorporates the possibility of an anomalous skin effect at the constriction.

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